In tali nunquam lassat venatio sylva.
A.D.1884.

AN

ARITHMETICAL GUIDE;

IN WHICH THE

PRINCIPLES OF NUMBERS

ARE

INDUCTIVELY EXPLAINED:

AND ALSO FAMILIARLY APPLIED TO

THE EVERY-DAY BUSINESS OF LIFE.

FOR SCHOOLS AND ACADEMIES.

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PREFACE.

In pursuing the business of instruction, so as to prepare my pupils for useful life, I have found the generality of school arithmetics to be, in many particulars, deficient for my purpose.

In the first place, many of the arithmetics that have fallen into my hands, serve to lumber the mind with much that is not only entirely useless, but that is also actually *in the way*, if I may so speak, in the business operations of mercantile or mechanical life. In the next place, they have not kept pace with the improvements of the age, and therefore lack much that it is highly necessary for the practical calculator to understand.

But this is not all. Every study should "call into exercise, and discipline and invigorate the powers of the mind." And for this purpose arithmetic is peculiarly adapted; especially if its principles be clearly explained and demonstrated, and so arranged as to show their dependence one upon another. But, in

the general method of teaching arithmetic, who has ever looked for a great change in the pupil's reasoning faculties? Let each one judge from experience. It is true, that the introduction of Colburn's works in many schools has had a redeeming effect in this particular. But, as his *Sequel* is thought by many to be too little practical, the good which is acquired from his *First Lessons*, has not been sustained in the pupil's after study, and has therefore been, in a great measure, lost.

On this account, I have endeavored to compile an arithmetic which shall be fitted, both to impart habits of just and rapid reasoning, and also to introduce the pupil into the operations of active life.

To accomplish these objects, it was necessary, in the first place, to present every principle in so simple a manner that the smallest mind can easily apprehend it. This has been done, progressively, by means of mental exercises, and inductive exercises for the slate. In these last, the rules in general use have been developed, by explaining and illustrating element after element, so as to make the pupil understand the philosophy of their origination; thus leading him to depend more upon scientific principles than on any set form of words. The exercises are followed by suitable definitions and rules, embracing the previously acquired information arranged in proper formulas. The pupil is then furnished with practical questions for the slate,

which he will find require no other operations than those with which he has already become familiar in his preceding exercises. While this course is pursued throughout the book, care has also been taken to form an arrangement of *general principles* in the most easy and natural order for the pupil's apprehension. And here the author would state that he considers the object of young pupils to be that of obtaining ideas; and that the business of systematic classification belongs to older scholars.

To make the book a practical work, it was necessary to adapt it to business as it is. This has been done, by making a great part of the operations, as it regards money affairs, in dollars and cents; omitting, in the part on compound numbers, calculations occasioned by such parts of tables as the following:—16 drachms make 1 ounce—3 barley corns make 1 inch—40 rods make 1 furlong—8 furlongs make 1 mile—40 square rods make 1 square rood—4 square roods make 1 square acre—63 gallons make 1 hogshead—4 weeks make 1 month—&c.; all of which are superfluous in business operations. We have also frequently reckoned by hundreds, instead of cwt., to meet the custom that is now adopted in our cities. Besides this, we have omitted a great part of Practice, as well as Tare and Tret, which are entirely useless in this country; and have facilitated many calculations by

imparting a perfect knowledge of Fractions. Proportion, or Rule of Three, is so explained as to do away the perplexity of two different methods of operation; Interest, Insurance, Duties, Exchange, &c., are explained as transacted in mercantile life; Extraction of Roots is explained by diagrams; and many other improvements of minor importance are interspersed throughout the work. That the book has some defects is probable; but the author is encouraged to believe that it is well fitted to prepare young men for actual business, and, at the same time, adapted for an introductory step to the higher branches of the Mathematics. R. W. G.

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TO TEACHERS.

It seems proper to state that the author's intention in giving a formula, with the first answer under each of the rules, was to supply a model for the pupil to follow in reciting the operation of his sums. Much of his advancement will depend on this recitation, whether in a class or by himself. In the Mental Exercises, especially, the whole operation should be given. Much improvement will be derived, also, if the class or the teacher will explain, on the blackboard, the sums embraced in the Inductive Exercises for the Slate.

ARITHMETIC.

In Arithmetic we learn the true method of calculating by numbers.

Its leading rules are Addition, Subtraction, Multiplication, and Division.

ADDITION.

Addition is the putting together two or more numbers into one.

INDUCTIVE EXERCISES FOR THE SLATE.

Section I.

	[1st sum.]	[2d sum.]	[3d sum.]	[4th sum.]
Add	36	467	548	760
and	42	511	420	124
The answer is	78	978	968	884

[5th sum.]	[6th sum.]	[7th sum.]	[8th sum.]
263	201	623	427
624	784	146	522

[9th sum.]	[10th sum.]	[11th sum.]	[12th sum.]
846	420	243	621
122	246	412	346
31	231	323	101

Section II.

☞ Hereafter the pupils should read their sums as an exercise in numeration; and then should *recite their work* as follows:

First part.	*13th sum. Add* 96 *and* 35.
96	6 *and* 5 *are* 11, *set down the* 11.
35	(It must be set down as in the first part.)
11	
Second part.	Then,
96	9 *and* 3 *are* 12; *and the* 1 *that is under it*
35	*makes* 13. *Rub out that* 1, *and set down* 13.
Ans. 131	*The whole answer is* 131.

The next five sums are begun. The pupil must finish them, by rubbing out the 1 that is under the column to be added.

[14.]	[15.]	[16.]	[17.]	[18.]
87	36	47	38	45
78	64	75	96	99
15	10	12	14	14

The pupil may now do the whole sum in the following.

[19.]	[20.]	[21.]	[22.]	[23.]
68	74	84	75	49
68	97	48	75	49

[24.]	[25.]	[26.]	[27.]	[28.]
98	36	44	77	99
36	36	88	66	44
22	94	55	75	55

[29.]	[30.]	[31.]	[32.]	[33.]
48	39	43	56	21
54	67	89	47	35
12	11	23	83	49

Section III.

[34.]
67
43
35
145

5 and 3 are 8, and 7 are 15; set down 15. The 1 of the 15, and 3 above it, are 4; and 4 are 8; and 6 are 14. Rub out the 1 and set down 14. The whole answer is 145.

[35.]	[36.]	[37.]	[38.]	[39.]
37	36	48	54	23
25	71	93	67	15
31	24	82	23	94
26	35	35	81	27

[40.]	[41.]	[42.]	[43.]	[44.]
61	72	52	36	41
29	23	24	83	19
37	97	46	87	28
63	62	37	42	34
21	49	88	79	46

[45.]	[46.]	[47.]	[48.]	[49.]
23	37	88	98	43
62	29	47	46	26
97	24	37	99	93
31	38	29	64	23
29	46	12	21	27
64	78	34	87	19

[50.]	Ans. 6 *and* 4 *are* 10, *and* 8 *are* 18. *Set*
Add 128	*down* 18. 1 *carried to the* 7 *above it makes*
904	8, (*the cipher is not added,*) *and* 2 *are* 10.
576	*Set down* 10. 1 *carried to the* 5 *above it are*
1608	6, *and* 9 *are* 15, *and* 1 *are* 16. *Set down*
	16. *The whole answer is,* 1608.

[51.]	[52.]	[53.]	[54.]	[55.]
126	196	478	201	869
932	302	43	390	602
327	234	680	49	84
709	408	877	924	224

[56.]	[57.]	[58.]	[59.]	[60.]
327	341	318	363	395
496	12	640	914	261
80	965	235	507	71
53	803	97	24	304
207	700	401	137	896

Section IV.

The pupil should now be able to add rapidly as follows:

[61.]	
1460	6...10...17...18. *Set down* 8, *and carry the* 1
2751	*before it.*
3027	1...10...12...17...23. *Set down* 3 *and carry the*
6894	2 *before it.*
2406	2...6...14...21...25. *Set down* 5, *and carry the* 2.
	2...4...10...13...15...16. *Set down the whole* 16.
	The whole answer is ——.

☞ For reading numbers, if there are more than three figures, the pupil must cut off the last three; and call the others *thousands.* Thus, 16,538; 16 *thousand, &c.*

[62.]	[63.]	[64.]	[65.]
2765	17095	14481	17422
1448	40462	77017	24704
7017	41025	40040	35400
402	87236	4196	8681
9623	4481	23875	49711
5256	71074	25614	70654

[66.]	[67.]	[68.]	[69.]
3296	2540	17162	4865
12314	17944	4815	27542
77458	2706	3072	377
90474	80344	58432	8468
46762	7111	781	39786
8152	57651	495	414
40114	48422	62549	2761

[70.]	[71.]	[72.]	[73.]
214	420	481	263
4866	3984	3984	41
37618	24610	24610	3946
4783	142	742	79
312	8976	18976	20421
53280	46525	525	6496
7870	240	1825	827

SUBTRACTION.

SUBTRACTION is the taking of one number from another, in such a manner as to show what is left.

INDUCTIVE EXERCISES FOR THE SLATE.

SECTION I.

[1st sum.]

From 137
take 14
123

Subtract 14 from 137.

4 *from* 7 *leaves* 3.
1 *from* 3 *leaves* 2.
Nothing from 1 *leaves* 1. *The answer is* 123.

[2d sum.]	[3d sum.]	[4th sum.]	[5th sum.]
238	3249	2366	1656
13	125	164	422

[6.]	[7.]	[8.]	[9.]
24287	18976	4695	8487
4023	1062	1525	6002

[10.]	[11.]	[12.]	[13.]
19876	28968	59047	89643
6143	1263	21012	23141

Section II.

[14.]
42
27
15

7 from 2, we cannot. We borrow 1 ten from the 4 tens; which added to the 2, makes 12. Then, 7 from 12 leaves 5. 2 from 3, leaves 1. The remainder is, 15.

or,

42
27
15

7 from 2 we cannot. We borrow 1 ten from the 4 tens; which added to the 2 makes 12. Then, 7 from 12 leaves 5. The 1 that we borrow added to the 2, makes 3; and 3 from 4 leaves 1. The answer is, 15.

[15.]	[16.]	[17.]	[18.]	[19.]
293	3327	4596	1471	574
176	142	278	325	456

[20.]	[21.]	[22.]	[23.]	[24.]
827	6426	765	7843	8765
765	219	466	744	378

[25.]	[26.]	[27.]	[28.]
3547	13849	4762	23256
2654	2956	2685	1189

[29.]	[30.]	[31.]	[32.]
34574	74596	16825	45869
3289	4278	3147	14870

[33.]	[34.]	[35.]	[36.]
48769	25872	48327	59627
39890	18987	19638	29548

Section III.

[37.]

407
268
139

As 8 *from* 7, *we cannot; we borrow a* 10 *from the* 40; *which added to* 7, *makes* 17. *And* 8 *from* 17 *leaves* 9. *We shall now call the* 40, *thirty-nine.* 6 *from* 9 *leaves* 3. 2 *from* 3 *leaves* 1. *The answer is* 139.

or,

407
268
139

8 *from* 7, *we cannot. We borrow* 1; *and* 8 *from* 17 *leaves* 9. 1 *to the* 6 *makes* 7; *and* 7 *from* 0, *we cannot. We borrow* 1 *ten from the* 4; *and* 7 *from* 10 *leaves* 3. 1 *added to the* 2 *is* 3; *and* 3 *from* 4 *leaves* 1.

[38.]	[39.]	[40.]	[41.]
37048	36052	40302	50400
13739	22774	28654	26244
[42.]	[43.]	[44.]	[45.]
40800	80200	97000	64000
25756	76732	6273	8765
[46.]	[47.]	[48.]	[49.]
27004	46200	10000	40020
15092	20009	909	37924

MULTIPLICATION.

MULTIPLICATION teaches how to find what number would be made, by adding a given number together several times.

Note.—The following *table* must be learned *perfectly*, before the exercise can be performed.

MULTIPLICATION TABLE.

2 times 1 are 2	3 times 1 are 3	4 times 1 are 4
2 — 4	2 — 6	2 — 8
3 — 6	3 — 9	3 — 12
4 — 8	4 — 12	4 — 16
5 — 10	5 — 15	5 — 20
6 — 12	6 — 18	6 — 24
7 — 14	7 — 21	7 — 28
8 — 16	8 — 24	8 — 32
9 — 18	9 — 27	9 — 36
10 — 20	10 — 30	10 — 40
11 — 22	11 — 33	11 — 44
12 — 24	12 — 36	12 — 48

5 times 1 are 5	6 times 1 are 6	7 times 1 are 7
2 — 10	2 — 12	2 — 14
3 — 15	3 — 18	3 — 21
4 — 20	4 — 24	4 — 28
5 — 25	5 — 30	5 — 35
6 — 30	6 — 36	6 — 42
7 — 35	7 — 42	7 — 49
8 — 40	8 — 48	8 — 56
9 — 45	9 — 54	9 — 63
10 — 50	10 — 60	10 — 70
11 — 55	11 — 66	11 — 77
12 — 60	12 — 72	12 — 84

8 times 1 are 8	9 times 1 are 9	10 times 1 are 10
2 — 16	2 — 18	2 — 20
3 — 24	3 — 27	3 — 30
4 — 32	4 — 36	4 — 40
5 — 40	5 — 45	5 — 50
6 — 48	6 — 54	6 — 60
7 — 56	7 — 63	7 — 70
8 — 64	8 — 72	8 — 80
9 — 72	9 — 81	9 — 90
10 — 80	10 — 90	10 — 100
11 — 88	11 — 99	11 — 110
12 — 96	12 — 108	12 — 120

11 times 1 are 11	12 times 1 are 12
2 — 22	2 — 24
3 — 33	3 — 36
4 — 44	4 — 48
5 — 55	5 — 60
6 — 66	6 — 72
7 — 77	7 — 84
8 — 88	8 — 96
9 — 99	9 — 108
10 — 110	10 — 120
11 — 121	11 — 132
12 — 132	12 — 144

How many are seven times 9? Nine times 6? Six times 4? Four times 7? Seven times 11? Eleven times 9? Nine times 2? Two times 8? Eight times 12? Twelve times 6? Six times 3? Three times 10? Ten times 5? Five times 6? Six times 7? Seven times 7? Seven times 5? Five times 4? Three times 5?

How many are four times 12? Twelve times 3? Three times 4? Four times 11? Eleven times 5? Five times 8? Eight times 9? Nine times 4? Four times 2? Two times 10? Ten times 7? Seven times 3? Three times 2? Two times 5? Five times 12? Twelve times 10? Ten times 9? Nine times 3?

How many are six times 8? Two times 7? Seven times 8? Eight times 4? Four times 10? Ten times 6? Six times 6? Six times 11? Eleven times 12? Twelve times 7?

How many are nine times 7? Six times 9? &c. saying each question backwards.

INDUCTIVE EXERCISES FOR THE SLATE.

Section I.

1. Multiply 48 by 3.

48 3 144	Ans. 3 *times* 8 *is* 24. *Set down* 4, *and carry* 2. 3 *times* 4 *is* 12, *and the* 2 *that was carried make* 14. *The answer is* 144.

	[2d sum.]	[3d sum.]	[4th sum.]
Multiply	13	34	42
By	3	2	2

	[5.]	[6.]	[7.]	[8.]
Multiply	123	323	421	321
By	3	3	4	5

	[9.]	[10.]	[11.]	[12.]	[13.]
Multiply	432	4321	3223	5432	3454
By	4	5	4	5	4

	[14.]	[15.]	[16.]	[17.]
Multiply	5423	3421	6435	5342
By	6	7	6	7

	[18.]	[19.]	[20.]	[21.]
Multiply	7465	6534	4523	5764
By	6	7	8	7

	[22.]	[23.]	[24.]	[25.]
Multiply	7465	4321	8576	6534
By	8	9	8	9

In the following sums, and throughout the arithmetic, the pupil must remember to begin to set down the answer *directly* under the figure which he *multiplies by*.

	[26.]	[27.]	[28.]	[29.]
Multiply	9756	7465	9876	9675
By	8	9	8	9

Section II

[30.]
5
70
350

Multiply 5 by 70.

Ans. 7 *times* 5 *is* 35. *Set down* 35 *under the* 7; *and put a cipher after it for the cipher that is after the* 7. *The answer is* 350.

	[31.]	[32.]	[33.]	[34.]
Multiply	8	27	250	460
By	900	3000	500	600
	7200	81000	125000	276000

The pupil may perform the following sums in the same manner; taking care to put the *last number* in the question underneath the number which is to be multiplied; and to put the first figure in the *answer* under the figure he multiples by.

35. Multiply 4 by 80.
36. Multiply 9 by 80.
37. Multiply 11 by 90.
38. Multiply 16 by 30.
39. Multiply 22 by 40.
40. Multiply 341 by 50.
1. Multiply 123 by 400.
42. Multiply 4767 by 60.
43. Multiply 241 by 700.
44. Multiply 476 by 800.
45. Multiply 395 by 400.
46. Multiply 462 by 9000.
47. Multiply 621 by 4000.
48. Multiply 849 by 7000.

Section III.

[49.]

18
14
72
18
252

Multiply 18 by 14.

Ans. 4 *times* 8 *are* 32; *set down* 2 *under the* 4. 4 *times* 1 *are* 4, *and* 3 *to carry are* 7. 1 *time* 8 *is* 8; *set down the* 8 *under the* 1. 1 *time* 1 *is* 1. *Both answers added together equals* 252.

	[50.]	[51.]	[52.]	[53.]
Multiply	14	121	192	146
By	17	69	34	28
	98	1089		
	14	726		
	238	9349		

Perform the following in the same manner.

54. Multiply 27 by 43.
55. Multiply 146 by 75.
56. Multiply 725 by 98.
57. Multiply 4760 by 59.
58. Multiply 305 by 47.
59. Multiply 5062 by 71.
60. Multiply 2477 by 86.
61. Multiply 3941 by 89.

[62.]	[63.]	[64.]
3246	9872	1875
304	407	306
12984		
9738		
986784		

DIVISION.

Division is the operation of finding *how many times* one number can be taken from another.

Section I.

When the divisor is not more than 12.

[1st sum.]	[2d sum.]	[3d sum.]	[4th sum.]	[5th sum.]
5)10	5)30	6)42	8)72	7)56
2	6	7		

[6.]	[7.]	[8.]	[9.]	[10.]
9)63	8)40	9)36	8)64	6)48

[11.]
4)840
210

Ans. *4 is contained in* 8, *2 times. 4 is contained in* 4 *one time. 4 is contained in* 0, *no time. The answer is* 210.

[12.]	[13.]	[14.]	[15.]
4)2448	3)3690	6)4860	8)6408
612			

[16.]	[17.]	[18.]	[19.]
5)2550	6)30612	8)56248	7)35742

Section II.

[20.]
6)1476
246

6 is contained in 14, *2 times and* 2 *over. The* 2 *that is over is put before the* 7 *and makes* 27. *6 is contained in* 27, *4 times and* 3 *over. The* 3 *is put before the* 6 *and makes* 36. *6 is contained in* 36, *6 times.*

[21.]	[22.]	[23.]	[24.]
2)7652	3)6789	3)6564	4)5736
[25.]	[26.]	[27.]	[28.]
5)6340	6)7776	6)5472	7)9471
[29.]	[30.]	[31.]	[32.]
8)9784	9)8757	7)87647	8)94328
[33.]	[34.]	[35.]	[36.]
9)43272	8)64248	9)75906	7)46235

Section III. Long Division.

When the divisor is more than 12.

[37.]

```
23)476432(20714
  46
  ---
  164
  161
  ---
   33
   23
   --
   102
    92
    --
    10 rem.
```

23 is contained in 4, no time. 23 is contained in 47, 2 times. Twice 23 is 46, which subtracted from 47, leaves 1. Instead of putting 1 before the 6, put 6 after the 1, which makes 16. 23 in 16, no time; bring down the 4 after it, and it makes it 164. 23 in 164, 7 times; and 7 times 23 is 161, which subtracted from 164, leaves 3. Bring down the next figure: and 23 in 33 one time, and 10 over. Bring down the next figure; and 23 in 102, is 4 times and 10 over. The whole answer is, 20714 and 10 remainder.

[88.]

44)74672(1
44
306

If the pupil wishes to finish this sum, he may try the *first* figure of the divisor. As he leaves *one* figure of the divisor, he must also leave one of 306. Therefore 4 in 30, 7 times. Put 7 in the answer, and 7 times 44 is 308. But as 308 cannot be subtracted from 306, 7 is too much for the answer. Rub it out and put down 6.

The pupil may be told that this method is the best he can pursue, though he will not always get right the first time. He may also be told that he must never put more than 9 at any one time in the answer.

39. Divide 276476 by 73.
40. Divide 123456 by 85.
41. Divide 341562 by 96.
42. Divide 452678 by 179.
43. Divide 213467 by 78.
44. Divide 245163 by 91.
45. Divide 123544 by 356.
46. Divide 564157 by 189.
47. Divide 567664 by 518.
48. Divide 246851 by 467.

ARITHMETIC.

NOTATION AND NUMERATION.

1. Every thing which exists of itself may be increased or diminished in two different ways. One of these ways is by varying the *collection of several similar things*; as in the examples of *one* apple, *two* apples, a *hundred* apples, &c. The other way is to alter the extent; that is to change the *length*, *breadth*, or *thickness* of the *same thing*.

Any thing which can be increased or diminished has received the general name of *quantity*.

2. *Number* is the particular name that is given to quantity in *collection*.

3. *Magnitude* is the particular name we give to quantity in *extent*.

Thus, a *line* is a magnitude, because it has length; any *surface* is a magnitude, because it has length, and breadth; any *solid* is a magnitude, because it has length, breadth, and thickness. Besides these, there are several other species of quantity; such as *weight*, *time*, *motion*, &c.

4. The first *idea* of *number* is formed by joining one thing to another; and then joining another to these two; and then to this collection, another; and so on as long as we choose.

---

1. In how many ways are objects susceptible of increase and diminution? Mention one way. Mention the other. What general name has been given to the property of increase and diminution? 2. What is number? 3. What is magnitude? Give some examples of magnitude. 4. How do we first obtain an idea of number?

5. In order to form an idea of *magnitude*, we first adopt any one particular extent, for a *measure*; and then, using that measure as a term of comparison, we ascertain what is the proportion between that and any other extent of the same species.

Thus, if I wish to ascertain the length of a given line, I first fix upon some known length, as a *foot* or an *inch*, as a measure; and then determine how many times this measure is contained in the proposed line. For example, the measure may be contained five times; and then the proportion of the required magnitude to the measure, is expressed by saying it is five times as large; or in common language, by saying it is *five feet* or *five inches long*.

6. Magnitudes are expressed with numbers by considering a collection of *several parts of the same quantity*, instead of a collection of *several similar things*.

Hence, *all kinds of quantities may be expressed with numbers*, by considering them, either as collections of several similar things, or as collections of several distinct parts.

7. As in most of our concerns we consider things according to their *number*, we are frequently obliged to *calculate* what effect will be produced by combining two or more numbers in any particular manner.

8. An examination into the different methods of calculating, constitutes the science of *Arithmetic*.

9. DEFINITION.—*Arithmetic is the science of numbers, and the art of calculating. As a science, it considers the nature and properties of numbers. As an art, it teaches how to represent numbers by figures, and furnishes easy and certain methods for operating with them.*

---

5. How do we form a definite idea of magnitude or size? Give an illustration. 6. How is the measure of magnitude expressed? How can we apply number to a magnitude? 7. What often happens from the fact that we have so much to do with number? 8. What do we call a description of the methods of calculating? 9. Define *Arithmetic*.

10. The first object of arithmetic is to establish a method of writing numbers in an abridged form; so that they may be more easily perceptible to the mind, and more readily used in our operations. This branch of arithmetic is called *Numeration*.

11. DEFINITION.—*Numeration is the art of writing any number by the aid of ten characters or figures.* It also embraces the ability to read any number when so written.

12. We are enabled to write so many different numbers with so small a variety of figures, by arranging those figures according to a system analagous to that of the numeral nomenclature.

13. The following are the figures and their names:

Cipher or nought,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.
0	1	2	3	4	5	6	7	8	9

All of these figures but the cipher are called *significant* figures. The *cipher* is *not* a significant figure, because it does not represent any number.

14. When either of these figures is used alone, it is called so many *units*. Thus, the figure 8 stands for eight units.

15. The meaning of *unit, is a single whole thing.*

16. We have no figure for a number over nine. Therefore, to represent more than nine, we make a figure stand for so many *collections*, instead of so many single things. Thus, we say 4 tens or forty; 5 hundreds; 7 thousands, &c.

17. This may be explained as follows:

In the first place, we know that, in our numeral nomenclature, the first order of names represents *ones* or *units*;

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10. What is the first object of arithmetic? Why is this necessary? 11. Define Numeration. 12. How can we write all our numbers with only ten figures? 13. Which are the *significant* figures? 14. When do these figures stand for *ones*? 15. What is the meaning of *unit*? 16. How can we make a figure stand for more than 9? 17. What is represented by the first order of names in counting? the second?

that the second order represents collections of *ten ones*, (as in *sixty*, meaning *six tens*;) that the third order or hundreds represents collections of *ten tens*; that the fourth order or thousands represents collections of *ten hundreds*; each name representing collections ten times as great as the next smaller one.

Accordingly, *the principle has been established, that after any name has been expressed by a figure, we may express the name of the next higher order by putting a figure immediately to the left of the figure already written.* Thus, 5 by itself represents five *units*; 5 immediately before another figure, represents 5 tens or fifty; as in 54, (*fifty and four*;) 5 immediately before tens, will represent 5 *hundreds*, as in 543, (*five hundred forty and three*;) 5 immediately before hundreds, will represent 5 *thousands*, as in 5627, (*five thousands, six hundreds, twenty and seven*;) &c.

It remains now to show how to write a number that is composed of some higher collections, but is lacking in some of the less; such as *sixty*, which has no units; six hundred and one, which has no tens; six thousand and seventy, which has no hundreds and no units. To write such numbers, it is evident that our only trouble will be to make each order hold its proper place. To effect this, if there are no units in the number, we fill the units' place with the cipher or 0, that the tens may be put before it; as in 60. If there are no tens, we fill the tens' place with a cipher, that the hundreds may be put before it; as 601. So if there are no hundreds, we fill the hundreds' place with a cipher; as in 6070. And in general, when in *enunciating a number, there has any order of names been omitted, the place of that order must be supplied with a cipher when the whole number is to be written in figures.*

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third? fourth? In counting, how much is one order of names greater than the preceding one? How can we express this in writing numbers? Repeat an example. How can we do this, when one or more of the intermediate orders is lacking?

Exercises.

1. What do we mean in saying *sixty-seven?*—Write it.

2. The same questions should be asked on each of the following numbers; which, when answered, should be written on the blackboard by the pupil.

Thirty-four; Seventy-six; Forty-eight; Ninety-three; Twenty-nine; Fifty-five; Eighty-one; Forty; Seventy four; Thirty; Forty-seven; Twenty; Sixty-nine

3. What do we mean in saying four hundred and sixty-five?—Write it.

4. Seven hundred and forty-two; Five hundred and thirty-six; Four hundred and sixty-four; Nine hundred and seventy-five; Two hundred and ninety-eight; Eight hundred and eighty-seven; Six hundred and twenty-five; Three hundred and eighteen; Seven hundred and sixteen; Four hundred and twelve; Eight hundred and seven; Three hundred and five; Seven hundred and four; Six hundred and three; Nine hundred and one.

5. What is meant by the number Four thousand five hundred and seventy-three? Ans. 4 thousands, 5 hundred, 7 tens, and 3 units.

6. Write it.

7. Each of the following may be repeated and written in the same manner.

Four thousand seven hundred and sixty-two.
Eight thousand four hundred and seventy-eight.
Nine thousand eight hundred and fifty-three.
Seven thousand and eighty-seven.
Six thousand four hundred and one.
Three thousand and six.
Four thousand and seventy-five.
Nine thousand.—Ten thousand.
Ten thousand and four hundred.
Ten thousand five hundred and forty-six.

Twelve thousand three hundred and four.
Fourteen thousand and fourteen.

The pupil must now understand that in reading numbers, we may say 14 *ones*; but that we cannot say 14 *tens*, or 14 *hundreds*. But arithmeticians *are* accustomed to say 14 *thousands*. In the same manner we can say 312 *ones*, but not 312 *tens*, or 312 *hundreds*. But we may say 312 *thousands*. On this account, we sometimes divide a large written number into periods of three figures each; and then the left hand period is called the *period of units*, and the *next period*, the *period of thousands*. Thus we may write 312 thousands and 312 as follows: 312,312.

23. The pupil may now mark the periods as he writes the following numbers. This will be done by writing first the number of thousands, and then taking care to make a comma and write after it three figures.

Sixty-one thousand. Ans. 61,000.
Ten thousand four hundred.
Twelve thousand six hundred and one.
Twenty thousand and three.
Forty thousand and forty.
Ninety thousand and nine hundred.
One hundred thousand.
Two hundred thousand and six hundred.
Three hundred and four thousand and four.
Four hundred and two thousand eight hundred.
Five hundred and fifty thousand and ten.
Six hundred and sixty thousand and fifteeen.
Seven hundred thousand four hundred and fifty.
Eight hundred thousand and forty-seven.
Nine hundred thousand and fourteen.
Seven hundred and one thousand and one.
Six hundred and thirty thousand and forty.

18. This method of writing numbers is made easy and expeditious by considering the following principle in our numeral nomenclature. In counting, our first three orders of names may be considered as *units*, tens of units, and hundreds of units; our next three orders may be considered as *thousands*, tens of thousands, and hundreds of thousands; the next three as *millions*, tens of millions, and hundreds of millions; the next three as *billions*, tens of billions, and hundreds of billions; and so on, making groups continually, each of three orders. Accordingly, when we wish to write a number, *we need only to consider the highest name mentioned, and write the number of that first,—which may take either one, two, or three figures; and then write the next highest name; and so on in succession, terminating at last with the units. And we must always remember that each name above* hundreds, *unless it be the very highest that is mentioned, will require a group of three orders, to express it.*

For example, if it be required to write forty billions, two hundred and seven millions, twenty-five thousands, four hundred and six, I would write it thus:

Billions, Millions, Thousands, Units.

40, 207, 025, 406, or 40,207,025,406.

19. It follows from this, that any number written in figures may be read in the following manner. *First, separate the number into periods of three figures each, proceeding from the right hand towards the left; and give them the names units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, &c. Then, commencing with the right hand period, read each division of figures as if it stood alone, and then add the name of the period.*

18. In numerating, how many figures may we consider as belonging to one group? Name the different groups, from the smallest to the greatest, and then from the greatest to the smallest. How does this assist us in writing large numbers? Give an example. 19. What rule is there for reading numbers?

For example, if we wish to read the number 7602410950051438; we will first point it off as follows, 7,602,410,950,051,438; and then read it thus; seven *quadrillions*, six hundred and two *trillions*, four hundred and ten *billions*, nine hundred and fifty *millions*, fifty-one *thousands*, four hundred and thirty-eight.

There was formerly a custom in England, which has been somewhat followed in this country, of giving the name of *billions* to millions of millions; of *trillions* to millions of billions, and so on. But the method here adopted of making periods of three figures instead of six, is universal on the continent of Europe, and is now adopted by the best mathematicians in England.

For some other directions, see page 206.

20. *To numerate* a number is to designate what order each of its figures represents, as follows:

7,	6	0	2,	4	1	0,	9	5	0,	0	5	1,	4	3	8
Quadrillions	Hundreds of Trillions	Tens of Trillions	Trillions	Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units

21. What is it to numerate a number? Numerate and read the following numbers.

1. 427654382.	6. 8900065431.
2. 565002761.	7. 28740062731.
3. 745260003.	8. 98200004.
4. 6028406.	9. 76542826841.
5. 94000786.	10. 3450876004.

Exercises.

Repeat and write the following numbers:

One million and forty thousand.

One million four thousand and thirty-two.

One million eight hundred thousand four hundred and two.

One million seven hundred and forty-one thousand eight hundred and seventy-nine.

Five millions four hundred and fifty-one thousand two hundred and thirty-two.

Ten millions seven hundred and thirty-six thousand eight hundred and eighty-one.

Eighteen millions four hundred and thirty-two thousand seven hundred and forty-six.

Twenty-one millions and six hundred thousand.

Forty-seven millions four thousand and four.

Ninety-six millions twenty thousand and seventy.

Seven hundred millions and four hundred thousand.

Six hundred and twenty millions and thirty.

Eight hundred and thirty-one millions and eighty.

Nine hundred millions and nine hundred thousand.

Four hundred millions four thousand and four.

MENTAL ADDITION,

Highly necessary to be understood by the pupil.

How much is 4 tens and 5 tens? How is it written?
How much is 7 tens and 2 tens? How is it written?
How much is 8 tens and 6 tens? How is it written?
How much is 7 tens and 9 tens when properly expressed?
How much is 4 tens and 8 tens when properly expressed?
How much is 6 tens and 7 tens when properly expressed? Then how much is 60 and 70?
How much is 7 tens and 8 tens when properly expressed? Then how much is 70 and 80?
How much is 3 tens and 4 tens when properly expressed? Then how much is 30 and 40?
How much is 20 and 60? 40 and 50?
How much is 30 and 30? 40 and 40?
How much is 50 and 70? 70 and 20?
How much is 40 and 80? 80 and 50?
How much is 70 and 90? 90 and 60?
How much is 4 hundred and 3 hundred?
How much is 600 and 300? 200 and 700?
How much is 500 and 400? 300 and 500?
How much is 400 and 600 when properly expressed?
How much is 600 and 600 when properly expressed?
How much is 700 and 600 when properly expressed?
How much is 700 and 800? 500 and 900?
How much is 600 and 800? 800 and 900?
How much is 4,000 and 5,000? 6,000 and 3,000?
How much is 9,000 and 8,000? 40,000 and 30,000?
How much is 300,000 and 500,000? 7,000,000 and 8,000,000?

ADDITION.

DEFINITIONS.

ADDITION is the putting together of two or more numbers in such a manner as to show how many they represent when combined.

That number which shows how many there are in two or more numbers when combined, is called the *sum* or *amount*.

The amount of small numbers may be found by adding them in the mind. But the amount of large numbers is found by the following

RULE.

First, State the question by putting units under units, tens under tens, &c. so that they will stand in columns.

Second, Begin at the right hand, and add up each column by itself; and write directly under it, its amount if less than ten. But if in writing the amount of any column, there should be more figures than one, set down the right hand figure only; because that belongs to the same order in numeration that the column does. But the left hand figure will represent the order of the next column, and must therefore be carried to that column.

What is *addition*? What is the *sum* or *amount*? What is it to state the question? What do you do when the amount of any column requires more than one figure to express it? But suppose the column is the farthest to the left? How do you prove addition?

Finally, Under the last column, write the whole of its amount.

PROOF.—The pupil may be taught that he may make himself more certain of a correct answer, by cutting off the top line, and finding the amount of the other lines; and then adding that amount and the top line together, in order to bring the whole amount. If the whole amount as found first, and the whole amount as found in this way, are alike, we say we have *proved the sum*.

Each of the following sums may be done, and afterwards proved.

SUMS FOR EXERCISE.

[1.]	[2.]	[3.]
80976431	1504036	60000000
7847365	64307	7000000
348000	4778977	499999
6749704	44499837	76874
8497460	4076088	4157843
104418960	54943245	71734716
23442529	53439209	11734716
104418960	54943245	71734716

[4.]	[5.]	[6.]
47065378	10007654	47600007
64673214	80706457	74800000
89674030	71424206	46500064
41071406	82634206	32100100
78600042	40810411	87643410

[7.]	[8.]	[9.]
12976534	1753962	1658173
76543129	6543179	7583942
54312976	5326474	5425813
31297654	1953632	3836352
97654312	7415196	9146187
65431297	3232751	6773952

[10.]
1753896
6548173
5823479
8350663
1908319

[11.]
1658170
7583094
5425030
3830085
8146380

[12.]
1753890
6548107
5823407
8350609
1003619

[13.]
1675558
8760037
2719976
3854021
6547382

[14.]
1823670
6778509
7615408
5094746
5090325

[15.]
16755587
60032719
97385473
87004200
9865616

[16.]
87659432
23495678
82736459
9546372
29753

[17.]
73298255
757337
34640049
66590
6240395

[18.]
57905900
5349940
230536
8760
49546402

[19.]
7890
76543
678901
6543210
56789
5432

[20.]
66577
4576
836500
85927
72944094
290130

[21.]
760740
91392077
6465
406
38
4980227

[22.]
675486
3920107
29385
4756
279630
57020949

[23.]
675609
80853
441
4909673
905722
20270

[24.]
93132
547
64867205
725090
780002
60099

[25.]	[26.]	[27.]
6547328	900674	500068
7534664	6556	904366
94640756	54340024	4726
457267	3774077	477267
207648	6276364	604
6034656	6626564	358466
76627	68764825	5675

[28.]	[29.]	[30.]
4680261	1030785	52448
79354680	68981632	698360
261793	67664	7107669
54680	904	53001702
2617	142578	2386453
935468	5650316	901618
2617935	32009	56432

[31.]	[32.]	[33.]
7908	57193427	716511
15794595	6767039	1668766
458474	799640	32890156
7783	9868	96556968
45352	83221	25935387
2873846	534175	07488476
26784638	7127737	14047296

APPLICATION.

1. A farmer sold his rye for 75 dollars; his corn for 84 dollars; his potatoes for 24 dollars; and his oats for 54 dollars. How many dollars did they all amount to? Ans. 237 dollars.

2. A gentlemen owed to one merchant 47 dollars; to another, 105 dollars; to another, 367 dollars; and to his tailor, 94 dollars. How much did he owe to them all? Ans. 613 dollars.

3. A person owns five farms. The first contains 475 acres; the second, 596 acres; the third, 874 acres; the fourth, 846 acres; and the fifth, 327 acres. How many acres do they all contain? Ans. 3118 acres.

4. A man paid at one time, 196 dollars for a chaise, and 175 dollars for a horse; and at another time, 60 dollars for a suit of clothes, and 47 dollars for other articles. How much did he pay in all? Ans. 478 dollars.

5. A man owns five houses. One is worth 1436 dollars; another, 1276 dollars; another, 1148 dollars; another, 1075 dollars; another, 975 dollars. What are they all worth? Ans. 5910 dollars.

6. Two farmers had each 475 acres of land. How many had they both? Ans. 950 acres.

7. Three teachers had each 47 scholars. How many scholars had they in all? Ans. 141 scholars.

8. A man had two sons and three daughters. He gave to his sons 240 dollars apiece, and to his daughters 225 dollars apiece. How many dollars did he give them all? Ans. 1155 dollars.

9. A merchant had three vessels, that were worth 6400 dollars apiece; a store worth 1500 dollars; and goods in the store worth 8600 dollars. How much did they all amount to? Ans. 29,300 dollars.

10. A butcher killed an ox, which was weighed in separate parts. Each of the two hind quarters weighed 175 pounds; each of the two fore quarters 152 pounds; the hide 105 pounds; and the tallow 108 pounds. What was the whole weight of the ox? Ans. 867 pounds.

11. From the creation of the world to the flood was 1656 years; from the flood to the Christian era was 2348 years; and from the Christian era till the time in which I am writing, is 1836 years. How long is it since the creation? Ans. 5840 years.

12. There are four loads of flour. The first contains 97 barrels, and is worth 776 dollars; the second contains 81 barrels, and is worth 630 dollars; the third contains 94 barrels, and is worth 704 dollars; and the fourth contains 77 barrels, and is worth 616 dollars. How many barrels of flour are there in the four loads, and what are they all worth? Ans. 349 barrels, and they are worth 2726 dollars.

13. Albert has 8 marbles, and James has twice as many. How many has James, and how many have they both?

Ans. James has 16 marbles, and both have 24.

14. A gentleman gave to one son 175 dollars, and to another twice as much. How many did he give to both?

Ans. 525 dollars.

15. James is 23 years old, and his father is 35 years older than he. How old is his father? Ans. 58 years.

16. A gentleman was requested to tell his age. He said that when he was married, he was 37 years old, and that he was married 49 years ago. How old was he?

Ans. 86 years.

17. At one o'clock, the clock strikes once; at two o'clock, the clock strikes twice, and so on till twelve. How many times does it strike from one to twelve? Ans. 78 times.

18. After twelve o'clock, time begins with one o'clock again. How many times does a clock strike in one day, or 24 hours? Ans. 156 times.

19. A man had six sons. He began with the youngest, and gave him 12 cents. To the next older, he gave 7 more than he did to the youngest. To the next, he gave 7 more, and so on, giving each 7 more than the last. How many did he give to the eldest, and how many to them all?

Ans. He gave 47 to the eldest, and 177 to them all.

20. A man bought a horse for 87 dollars, and was so fortunate as to sell him for 48 dollars more than he gave for him. For how much did he sell him? Ans. 135 dollars.

21. A merchant has due to him from people in Boston, 325 dollars in book accounts, and 756 dollars in notes; from people in New York, 826 dollars in book accounts, and 274 dollars in notes; and from people in Philadelphia, 675 dollars in notes, and 878 dollars in book accounts. How much is due to him on book account, and how much in notes? Ans. 2029 dollars on book account, and 1705 dollars in notes.

22. Peter gave his sister 27 chestnuts, and 25 walnuts; Robert gave her 37 walnuts, and 42 chestnuts; John gave her fifteen chestnuts and 34 walnuts; and Alfred gave her 45 walnuts and 37 chestnuts. Tell me how many walnuts and how many chestnuts she had, and how many nuts of both kinds each gave to her.

SIGNS.

The pupil may now be told, that this sign, + which is called *more*, or *plus* which is the Latin for *more*, is the sign of addition; that is, it shows that the number after it is to be added to the number before it. And two parallel lines are used for a sign of equality; that is, that the sums before it amount to the same number as the sums after it.

Thus, $4 + 3 = 7$, is read, 4 plus 3 is equal to 7; or 4 added to 3 is equal to 7.

Such expressions as $4 + 3 = 7$, are called *equations*. They are made up of quantities which are differently represented, and yet equal to one another. The pupil may now see whether the following equations are correct.

23. $7496 + 8765 + 426 + 8765 + 476 = 25928$.

24. $7848 + 4247 + 277596 + 666 + 6558 = 296915$.

25. $47 + 478 + 4789 + 47891 + 7891 + 891 + 91 = 62078$.

26. $123456789 + 12345678 + 234567 + 3456 = 136040490$.

27. Eight thousand four hundred + ninety-six thousand and seventy-five + fourteen thousand and fourteen + eighty-four thousand and twelve + seven hundred and six + four thousand eight hundred and thirty-seven + nine thousand and nine + six thousand and sixty = 223113.

28. Ninety-four thousand eight hundred and seventy-five + sixty-two thousand and sixteen + fourteen thousand and forty + eighteen hundred and seventy-two + nine millions one thousand and five + fourteen millions seventy thousand and seven + ten millions eight hundred and ninety = 33241705.

29. Four millions seven hundred and fifty-four + eighty thousand five hundred and six + four thousand eight hundred and seventy-five + eleven millions nine thousand and seventy-six + ten thousand four hundred and eleven = 15105622.

30. Sixteen millions + sixteen thousand and sixteen + sixteen hundred and sixty + sixteen and six = 16017698.

31. Eleven millions + eleven hundred and eleven thousand + eleven hundred + eleven = 11112111.

SUBTRACTION.

DEFINITIONS.

SUBTRACTION is the taking of one number from in such a manner as to show what number is left.

2. The number to be subtracted from is called the *minuend*; which signifies to be diminished.

3. That number which is to be subtracted is called the *subtrahend*; which signifies to be subtracted.

4. The number which is left after subtracting is called the *remainder*; and the remainder is the answer to the question.

5. When the numbers are small, their remainder may be found by subtracting in the mind. But when the numbers are large, we find the remainder by the following

RULE.

First, State the question, by placing the less number under the greater, taking care to put units under units, tens under tens, &c.; and then draw a line underneath.

Second, Beginning at the right, take each figure of the lower number from the figure above it, and write the remainders underneath the line.

But, if any upper figure is less than the figure below it.

What is *subtraction*? What is the minuend? What is the subtrahend? What is the remainder? What does the remainder show? How do we subtract small numbers? Can we subtract large numbers in the mind? How do you state a question in subtraction? How do you proceed after the sum is stated? Supposing any upper figure is less than the one under it? How many methods are there of subtracting the next figure, after we have borrowed? What is the most common? What is the other? How are sums in subtraction proved?

call it *ten more* than it is; and to compensate for this, proceed as if the next figure at the left was diminished by *one*.

Note.—There are two methods of proceeding when we suppose an upper figure to be diminished by one. The common method is to call it as it is written; and while we are subtracting from it, to add *one* to the under figure, so that the answer will be as is wanted, one less than what would arise from the written figures.—The other, and perhaps a better method, is actually to call the diminished figure one less than it is written; or, if it is a cipher, to call the next *significant* figure one less than it is, and to consider the cipher and also each other cipher until we come to that significant figure, as a 9.

Proof.—The correctness of the answer may be more relied upon, if after adding the *remainder* and the less number together, we find their sum to be equal to the greater number.

Each of the following sums may be performed, and then proved.

SUMS FOR EXERCISE.

	[1.]	[2.]	[3.]
Minuend	478947	984300	890079
Subtrahend	369474	497699	498746
Remainder			
Proof			

	[4.]	[5.]	[6.]
From	5648307	7038465	17627802
Take	2654328	2345628	9786546

	[7.]	[8.]	[9.]
From	50269278	76948007	29746627
Take	3476942	635498	18887738

	[10.]	[11.]	[12.]
From	39780062	28764900	17000605
Take	17896547	27857686	9870551

	[13.]	[14.]	[15.]
From	15487002	20706070	146767076
Take	8698117	19748091	56787178

	[16.]	[17.]	[18.]
From	12345678	82736450	80257436
Take	8765432	5463728	63475208

	[19.]	[20.]	[21.]
From	10283746	60427065	65064027
Take	6473820	56072406	7204605

APPLICATION.

☞ *After doing* 15 *of the following, the* YOUNG *pupil may commence with the remarks before the* 27*th question.*

1. WILLIAM has 54 cents, and John has 37. How many cents has William more than John? Ans. 17 cents.

2. Peter gave Samuel 93 marbles, and then had 48. How many marbles did he give Samuel more than he kept himself? Ans. 45.

3. A merchant bought 1476 pounds of coffee. After selling 987 pounds, how much had he left? Ans. 489 pounds.

4. A man has property worth 10796 dollars; but he owes 1847 dollars. How much is he worth?
Ans. 8949 dollars.

5. There are two numbers, 1843 and 2602. How much is one greater than the other? Ans. 759.

6. A man owing 1400 dollars, paid 490 dollars. How much did he continue to owe? Ans. 910 dollars.

7. A man owing 765 dollars, paid all but 78 dollars. How much did he pay? Ans. 687 dollars.

8. America was discovered in 1492. It is now 1836; how long is it since the discovery of America?
Ans. 344 years.

9. On new year's day a person had 4672 dollars. On the 4th of July he had 3785 dollars. How much had he expended? Ans. 887 dollars.

10. A man being asked how old he was when he was married, said that his present age was 84 years; and he was married 57 years ago. What was his age when he was married? Ans. 27 years.

11. A merchant bought a piece of cloth for 322 dollars, and sold it for 287 dollars. Did he gain or lose by the trade; and how much? Ans. Lost 35 dollars.

12. What number must be added to 4862 to make the sum of 5946? Ans. 1084.

13. To what number must 761 be added to make 1000000? Ans. 999239.

14. I bought 1324 yards of muslin for 257 dollars; and sold it for 212 dollars. Did I gain or lose, and how much?
Ans. Lost 45 dollars.

15. I bought 10000 pounds of coffee for 2760 dollars; and sold them for 3100 dollars. Did I gain or lose, and how much? Ans. Gained 340 dollars.

16. A man bought two hogsheads of molasses. One hogshead contained 73 gallons, and the other, 69 gallons. After he sold 105 gallons, how much had he left?

Ans. 37 gallons.

17. A man bought a house for 1225 dollars, but in selling it, is willing to lose 350 dollars. What is his price for it?

Ans. 875 dollars.

18. A man having 2000 dollars, paid one debt of 825 dollars, and another debt of 530 dollars. How many dollars had he left?

Ans. 645 dollars.

19. There was owing to a certain man one debt of 463 dollars, and another of 376 dollars; but he owed to one person 267 dollars, and to another, 525 dollars. How much more was owed to him than he owed to the others?

Ans. 47 dollars.

20. A farmer gave 2965 dollars for his house and his farm. The price of his house he called 925 dollars. What was the price of his farm without the house; and for how much more did he value it than the house?

Ans. 2040 dollars for the farm, and 1115 dollars more for it than for the house.

21. A man left at his death an estate worth 8975 dollars. In his will, he gave each of his two sons 1650 dollars; and to each of his three daughters, 1575 dollars; and the remaining part of it to his niece. How much did his niece receive; and how much more was given to one son than to one daughter?

Ans. His niece received 950 dollars; and a son's share was 75 dollars more than a daughter's.

22. A man once owed for his house 1500 dollars; for three acres of ground, 500 dollars; for his horse, 125 dollars; and for his carriage, 225 dollars. He paid at one time 600 dollars; at another, 750 dollars; at another, 425 dollars; and at another, 335 dollars. How much does he owe for them now?

Ans. 240 dollars.

23. A man has three bags of specie, containing 435, 523, and 325 dollars. He has also four parcels of bank notes, containing 400, 300, 325, and 237 dollars. How many dollars has he in specie; and how many more in specie than in notes?

Ans. He has 1283 dollars in specie, and 1262 dollars in bank notes. He has 21 dollars in specie more than he has in notes.

24. A person owed a merchant 475 dollars, and paid him 348. He afterwards borrowed of the same merchant 137 dollars. How much did he owe then? Ans. 264 dollars.

25. Bought 5 chests of tea, weighing 56, 61, 63, 58, and 59 pounds; and sold to one man 36 pounds, to another 27, to another 45, to another 50, and to another 96. How many pounds are there remaining? Ans. 43 pounds.

26. A person, when he commenced business, had 500 dollars in money, and 700 dollars in goods. Now, he has 500 dollars in money and 2500 dollars in goods. How much has he made in business? Ans. 1800 dollars.

The pupil may now be told that this sign, — which is called *less*, or *minus* which is the Latin for less; is the sign of subtraction. That is, it shows that the number after it, is to be subtracted from the number before it. Thus, $6 - 4 = 2$, is read, 6 minus 4 equals 2, or 6 less 4 equals 2, or 6 with 4 subtracted is equal to 2.

The pupil has heretofore written the subtrahend under the minuend; but in the following examples he must habituate himself to subtract without putting the subtrahend in that situation.

27. $484347 - 48434 = 435913.$

28. $794653 - 356497 = 438156.$

29. $1276543 - 1187654 = 88889.$

30. $9132 - 7583 = 1549.$

31. $27646 - 7987 = 19659.$

32. $35060 - 5076 = 29984$.

33. The sign + *plus*, is called by mathematicians, the *positive* sign; and the sign — *minus*, is called the *negative* sign. Accordingly, a number which has + *before* it, is called a *positive* number; and a number that has — before it, is called a *negative* number. If the first number in an equation has no — before it, it is positive.

The beginner in arithmetic may consider every equation as containing two sums; and he must understand that the answer of that on the *left* hand side of = is always the *same* as the answer of that on the *right* hand side.

When there are several positive and several negative numbers on the same side of =, add all the positive numbers together for one quantity, and all the negative numbers together for another quantity; and the *difference* between these two quantities will be the answer.

34. $786943 - 46732 - 23467 = 616744 + 100000$.

Negative quantities $\left\{\begin{array}{r} 46732 \\ 23467 \\ \hline 70199 \end{array}\right.$ Then $\left\{\begin{array}{r} +786943 \\ -\ 70199 \\ \hline 716744 \end{array}\right.$ Also, $\left\{\begin{array}{r} +616744 \\ +100000 \\ \hline 716744 \end{array}\right.$

Ans. $716744 = 716744$.

35. $46103 + 47310 + 347642 + 26434 = 467489$.

36. $118964 - 87642 - 27436 = 97463 - 93577$.

37. $147 + 465 - 276 - 274 = 467 + 576 - 981$.

38. $46539 + 6397 + 4276 + 1495 = 106341 - 47634$.

39. $4654 + 6544 - 7648 = 3550$.

40. $4652 - 7826 + 50437 - 2896 = 14785 + 29582$.

41. $14276 + 2864 + 3765 = 28397 - 7492$.

42. $37 + 429 + 1568 - 395 - 874 = 2264 - 765$.

What is a positive number? Why is it so called? What is a negative number? Why? Has every positive number + before it? How is the sum on the left hand side of = performed? How is that on the right? How will the answers compare?

MULTIPLICATION.

DEFINITIONS.

1. MULTIPLICATION is the operation of finding what sum is equal to a given number added together several times.

2. The number which is to be taken a given number of times, is called the *multiplicand;* which means *to be multiplied.*

3. The number that represents how many times another number is to be added together, is called the *multiplier;* Thus, when we wish to add 22 three times, 3 is the multiplier.

4. The sum or amount that is obtained by multiplying the multiplicand by the multiplier, is called the *product.*

5. The product of the multiplicand and the multiplier, is found by the following

RULE.

First, State the question, by writing down the multiplicand, and putting the multiplier under it, so that units shall be under units, tens under tens, &c.; and draw a line underneath.

Second, Begin with the units in the multiplier; and by it, multiply the units, and then the tens, &c. in the multiplicand; and set under each figure in the multiplicand, its product, taking care to carry, as in addition.

Third, If there is in the multiplier, a figure in the tens' place, multiply the multiplicand by that, taking care to set

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What is multiplication? What is the multiplicand? What is the multiplier? What is the product? How is multiplication performed? Supposing there are ciphers in the multiplier?

the first figure of its answer directly under the figure you multiply by.

Fourth, Proceed in the same manner with the hundreds, thousands, &c. of the multiplier; and finally, add the several products together.

When the multiplier terminates with one or more ciphers, it is necessary, before we multiply by the first significant figure, to write in the product as many ciphers as there are in the multiplier.

APPLICATION.

WITHOUT THE SLATE.

☞ The object of this exercise is to make the pupil very familiar with the method of answering. The printed answer must be *well committed to memory*. The same may be said of every model answer throughout the book.

1. What must you give for 4 barrels of flour, at 6 dollars a barrel? Ans. *Four barrels will cost four times as much as* 1 *barrel; and* 4 *times* 6 *dollars are* 24 *dollars.*

2. What cost 7 loads of hay, at 8 dollars a load?

3. If one cow costs 9 dollars, what will 8 cows cost?

4. If one pair of boots costs 7 dollars, what will 9 pair cost?

5. If one pound of pork costs 6 cents, what will 8 pounds cost?

6. What is the cost of 5 pounds of sugar at 8 cents a pound?

7. What is the cost of 9 melons at 4 cents a piece?

8. What must I give for 6 chairs at 4 dollars a piece?

9. If 1 ream of paper is worth 5 dollars, what is the worth of 7 reams?

10. If one inkstand costs 6 cents, what is the cost of 4 inkstands?

11. If I give 8 cents for one primer, what must I give for 4 primers?

12. If I travel 4 miles in one hour, how far will I travel in 7 hours?

13. What is the value of 8 pints of nuts at 3 cents a pint?

14. What is the value of 6 lead pencils at 7 cents a piece?

15. If a man earn 9 dollars in one month, how much will he earn in 7 months?

16. If my board is 4 dollars a week, what will 9 weeks' board come to?

17. If I earn 10 dollars a week, what do I earn in 9 weeks?

18. If 1 writing book costs 6 cents, what will 8 writing books cost?

FOR THE SLATE.

1. What costs 8 loads of hay at 13 dollars a load?

Ans. *If one load of hay costs* 13 *dollars*, 8 *loads will cost* 8 *times* 13 *dollars; which is*

13 dolls.
8
104 dolls.

2. What is the value of 7 dozen of quills at 25 cents a dozen?

Ans. 175 cents.

3. What is the cost of 6 oxen at 35 dollars a piece?

Ans. 210 dollars.

4. If one acre of land is worth 74 dollars, what is the worth of 7 acres of the same quality of land?

Ans. 518 dollars.

5. If I give 28 cents a pound for coffee, how much must I give for 18 pounds?

Ans. 504 cents.

6. What is the value of 462 bales of cotton at 46 dollars a bale?

Ans. *If one bale costs* 46 *dollars,* 462 *bales will cost* 462 *times* 46 *dollars; which is*

462
46 dollars.
2772
1848
21252 dollars.

☞ We know that 46 times 462, and 462 times 46 must make the same answer. Therefore, we put the smaller number underneath the larger, although in reading it, we say 462 times, &c.

7. What is the value of 894 acres of land at 94 dollars an acre? Ans. 84036 dollars.

8. There are 365 days in a year. How many days are there in 35 years? Ans. 12775 days.

9. How many days are there in 49 years?
Ans. 17885 days.

10. How many days are there in 82 years?
Ans. 29930 days.

11. How many days are there in 100 years?
Ans. 36500 days.

12. There are 24 hours in one day. How many hours are there in 34 days? Ans. 816 hours.

13. How many hours are there in 62 days?
Ans. 1488 hours.

14. How many hours are there in 303 days?
Ans. 7272 hours.

15. How many hours in 365 days or 1 year?
Ans. 8760 hours.

16. Sixty minutes make 1 hour. How many minutes in 24 hours or 1 day? Ans. 1440 minutes.

17. Sixty seconds make 1 minute. How many seconds in 45 minutes? Ans. 2700 seconds.

18. How many seconds in 60 minutes or 1 hour?
Ans. 3600 seconds.

19. If a man travels at the rate of 94 miles a day, how far will he travel in 24 days? Ans. 2256 miles.

20. What will 18 pounds of tea come to, at 108 cents a pound? Ans. 1944 cents.

21. There were 3 hogsheads of molasses, containing each 68 gallons; and 4 barrels of molasses, containing each 33 gallons. How many gallons in all?
Ans. 336 gallons.

22. A man has 4 pieces of cloth, each containing 31 yards; and 2 pieces containing each 37 yards; and 5 pieces containing each 34 yards. How many yards were there in all the pieces? Ans. 368 yards.

The pupil may now be told that this sign $\times$ when placed between two numbers, denotes that they are to be multiplied together. Thus, $6 \times 4 = 24$; is read, 6 multiplied by 4, equals 24.

23. $705684 \times 1763 = 1244120892$.

24. $6479 \times 109 = 706211$.

25. $647906 \times 4873 = 3157245938$.

26. $6765 \times 9000 = 60885000$.

27. 357269×235.

28. 819605×573.

29. 4076153×2085,

30. 2358746×7304.

FEDERAL MONEY.

INDUCTIVE EXERCISES FOR THE SLATE.

1. One hundred cents is called a dollar; how many dollars are there in 200 cents? How many in 300 cents?

2. How many dollars in 400 cents? In 600 cents?

3. How many dollars in 105 cents? In 176 cents?

4. How many dollars in 210 cents? In 292 cents?

5. How many dollars in 341 cents? In 536 cents?

The pupil may be told that if we wish to write 5 dollars, 36 cents, we put a dot between the dollars and the cents, and write this character $ before the dollars. Thus, $5.36. Four dollars may be written either $4. or $4.00; that is, 4 dollars no cents.

6. The pupil may now write the following: 5 dollars 31 cents;—4 dollars 41 cents;—9 dollars 10 cents;—7 dollars 9 cents.

The pupil must remember that dollars are in the *hundreds'* place of cents; and that therefore if we write cents after dollars, we must make *two* figures for them. The last answer should be written $7.09. Numerate it, and you will perceive the 7 is 7 hundred cents, which is 7 dollars.

7. Write 8 dollars 8 cents;—14 dollars 70 cents;—19 dollars 6 cents;—24 dollars 2 cents;—9 dollars 17 cents.

8. How many cents are equal to 4 dollars?—5 dollars?—6 dollars?—6 dollars 25 cents?—7 dollars?—7 dollars 50 cents?—8 dollars?—8 dollars 75 cents?—9 dollars 67 cents?—10 dollars 41 cents?

From these examples, it is very plain that the expressions $4.94, and $6.47, are the same as 494 cents, and 647 cents.

9. We sometimes say that a cent is as much as *ten mills*. How many mills are there in two cents?

10. How many mills are there in 3 cents? How many in 4 cents? How many in 6 cents?

The pupil will now understand that if he has any number of cents, it will consist of *ten times* as many mills. Therefore, *to reduce cents to mills*, he is only to *annex a cipher* to the number of cents. Of course to change mills back to cents, we must cut off the last figure.

11. How many mills in 8 cents? How many in 14 cents? How many in 18 cents?

12. How many cents in 30 mills? How many in 60 mills? How many in 80 mills?

13. How many mills in 9 cents? How many in 12 cents? How many in 21 cents?

14. How many mills are there in half a cent? How many in a cent and a half?

15. How many cents are there in 11 mills? How many in 13 mills? How many in 15 mills?

16. How many cents are there in 21 mills? How many in 33 mills? How many in 85 mills?

17. How many mills are there in 12½ cents? How many in 15½ cents?

18. How many mills in 20 cents? How many in 40 cents? In 90 cents?

19. How many cents in 100 mills? How many in 105 mills? In 125 mills?

20. How many cents in 200 mills? How many in 220 mills? In 275 mills?

21. How many cents are there in 405 mills? How many in 760 mills? In 995 mills?

22. How many mills in 100 cents or a dollar?

The pupil will now see that in mentioning the number of mills in a *dollar*, we use *four figures*; but in mentioning the mills in any number of cents *less* than a dollar, we

use but *three figures*, because if there is not a whole dollar, there will not be a thousand mills; and any number less than a thousand cannot have four figures; as may be seen in the number 999.

23. When we wish to change *dollars* to *mills*, how many figures must be written after the dollars? Ans. *Three.*

24. How many mills are there in 2 dollars? How many in 2 dollars 10 cents? In 2 dollars 50 cents?

25. We see that when there are *four figures* in any number of mills, the *thousandth's place* must be *dollars.* Now how can you *change* more than 1000 *mills* to *dollars?* Ans. *Cut off three of the right hand figures by a dot;* thus, 1565 mills are equal to $1.565.

☞ In reading such numbers as this, the *two* figures next to the dollars are called cents. Thus, 1 *dollar* 56 *cents* 5 *mills.*

MULTIPLICATION OF FEDERAL MONEY.

☞ The first thirty questions that are found in the former editions, are embraced in the preceding exercises.

1. How many cents are 4 times 67 cents? How many dollars is that? Ans. $2.68.

2. How many cents are 12 times 54 cents? How many dollars is that? Ans. $6.48.

3. How many cents are 9 times 167 cents? How many dollars is that? Ans. $15.03.

4. How many cents are 6 times $4.66? How many dollars is that?

☞ Whenever we wish to multiply a number consisting of dollars and cents, it is best to call it all cents; and then the answer will be in cents, which may after

wards be changed to dollars. It may be said as follows;

Ans. 6 *times* 466 *cents; which is* 2796 *cents; which equals* $27.96

$4.66
6
$27.96

5. How much is 4 times $7.45? Ans. $29.80.

6. How much is 7 times $9.27? Ans. $64.89.

7. How much is 9 times $12.37? Ans. $111.33.

8. How much is 325 times 47 cents?
☞ See page 53. Ans. $152.75.

9. How much is 627 times 28 cents? Ans. $175.56.

10. How much is 327 times 86 cents? Ans. $281.22.

11. How much is 245 times $3.40? Ans. $833.00.

12. How much is 462 times $4.45? Ans. $2055.90.

13. Write 25 dollars 14 cents 1 mill. How many mills in it all? How many mills are 7 times that? How many dollars and cents is it equal to?

☞ Whenever we wish to multiply a quantity, in which there are dollars, cents, and mills; it is best to call them all mills. And then the answer will be in mills; which may afterwards be changed to dollars and cents. Therefore,

Ans. 7 *times* 25141 *mills, which is* 175987 *mills; which equals* 175 *dollars*, 98 *cents, and* 7 *mills.* $25.141
7
$175.987

14. How much is 4 times $5.775? Ans. $23.100.

As there are no mills in the last answer, we may omit the figure that stands for mills; and write the answer thus, $23.10.

15. How much is 7 times $36.666? Ans. $256.662.

16. How much is 8 times $42.375? Ans. $339.00.

17. How much is 3 times 37 cents and 5 mills?
Ans. $1.125.

18. How much is 8 times 87 cents and 5 mills?
Ans. $7.

19. How much is 12 times 47 cents 8 mills?
Ans. $5.736.

20. How much is 16 times $4.237? Ans. $67.792.

21. How much is 15 times $3.486? Ans. $52.29.

22. How much is 21 times 9 dollars, 10 cents, and 3 mills? Ans. $191.163.

23. How much is 35 times 14 dollars, 12 cents, and 5 mills? Ans. $494.375.

24. How much is 46 times 127 dollars, 14 cents, and 8 mills? Ans. $5848.808.

25. How much is 58 times 97 dollars, 06 cents, and 7 mills? Ans. $5629.883.

26. How much is 76 times 84 dollars, 8 cents, and 2 mills? Ans. $6390.232.

27. How much is 82 times 123 dollars, 5 cents, and 6 mills? Ans. $10090.592.

28. How much is 68 times 14 dollars and 5 mills?
Ans. $952.34.

29. Multiply 37 dollars 8 mills, by 35.
Ans. $1295.28.

30. Multiply 46 dollars 2 mills by 27.
Ans. $1242.054.

31. Multiply $25.141 by 7. Ans. $175.987.

32. How much is 4 times $5.775? Ans. $23.10.

33. What will 57 yards of velvet come to, at $2.25 a yard? Ans. $128.25.

34. What is the value of 112 pounds of coffee, at 21 cents a pound? Ans. $23.52.

35. What is the cost of 38 pieces of linen, at $16.50 a piece? Ans. $627.00.

36. If 23 men receive 112 dollars a piece, how much will they all receive? Ans. $2576.

37. If one barrel of cider costs $2.75, how much will 37 barrels cost? Ans. $101.75.

38. A man purchasing several articles of a merchant, told him if he would give him a bill of them, he would

pay the cash for them. The merchant gave him the following. Did he make it out right?

Philadelphia, March 28, 1835.

James Paywell

Bought of Charles Fairprice.

21 yards of	Sheeting	at	19 cts.	a yard - -	$3.99
3 " "	Linen	"	37 "	" - -	1.11
10 " "	Broadcloth	"	6.50 "	" - -	65.00
2 Vest Patterns		"	1.25 "	each - -	2.50
					$72.60

Received payment,

Charles Fairprice.

As Mr. Paywell paid for the articles, Mr. Fairprice wrote *received payment*, and signed his name, as you see under the bill. This is called *receipting the bill*, which every one who pays for his articles should be careful to see done.

39. A purchaser of several articles requested a bill of them, stating that he would pay for them in a few days. It was made out as follows. Is it correct?

New York, April 30, 1836.

Henry Trustworthy

Bought of Timothy Givecredit.

21 pounds of	Sugar	at	9 cents	a pound - - -	$1.89
8 "	Coffee	"	27 "	" - - -	2.16
12 gallons of	Molasses	"	37 "	a gallon - - -	4.44
9 pounds of	Raisins	"	14 "	a pound - - -	1.26
2 Nutmegs		"	5 "	each - - -	10
					$9.85

As the bill was not paid, Mr. Givecredit did not receipt it.

40. A person wished to trade with a merchant on account of another person, who owed him. The following bill was made out. See if it is correct.

Baltimore, June 1, 1834.

William Daylaborer

Bought of Frederick Trader.

2 pairs of Shoes	at $2.00 a pair - - - - -	$4.00
3 pint Bowls	" 6 cents - - - - - - -	18
1 stone Jug	" - - - - - - - - - -	25
½ quire of Paper	" - - - - - - - - - -	13
4 pounds of Sugar	" 10 cents - - - - - - -	40
		$4.96

Received payment by discount with S. Farmer,

Frederick Trader.

41. Complete the following bill.

Philadelphia, Jan. 8, 1836.

Robert Buyer

Bought of Jonas Wiseman.

35 pounds of	Sugar	at	8 cents	a	pound	-
14 "	Coffee	"	28 "		"	-
6 "	Raisins	"	16 "		"	-
11 "	Loaf Sugar	"	20 "		"	-
2 gallons of	Molasses	"	34 "	a	gallon	-
						$10.56

Received payment by his note of the above date,

Jonas Wiseman.

42. Make out a bill of the following articles, dating it at your own residence, and at the time you write it.—Mr. Samuel Purchase bought of Mr. John Seller, 9 pounds of tea at 84 cents a pound; 11 pounds of sugar at 9 cents a pound; 2 quarts of molasses at 10 cents a quart; 2 yards of cloth at $5.25 a yard; and 14 yards of muslin at 21 cents a yard. For which he paid $22.19. Is it right?

43. A man bought 2341 pounds of coffee, at 15 cents a pound; and sold the whole of it for $384.10. Did he lose or gain; and how much? Ans. He gained $32.95.

44. A man bought 120 yards of muslin, for $25.50; and sold it for 23 cents a yard. How much did he gain?
Ans. $2.10.

45. A man bought 427 bushels of corn, at 37 cents a bushel; and sold it for $175.25. How much did he gain?
Ans. $17.26.

46. A man bought 326 yards of cloth, at $5 a yard; and sold it at $6 a yard. How much did he gain? Ans. $326.

47. A man bought 5 chests of tea, each chest containing 78 pounds, at 87 cents a pound. How many pounds did he buy; and how much did it all cost; and what did he gain if he sold it at 93 cents a pound?
Ans. He bought 390 pounds; and it cost him $339.30; and he gained $23.40.

48. I bought 894 yards of cloth, for $4470; and sold it for $7 a yard. What did I gain? Ans. 1788 dollars.

49. A man bought 4 pieces of cloth, each piece containing 31 yards, at $5.50 a yard; and sold it for $226.00 a piece. Did he lose or gain; and how much?
Ans. Gained $222.00.

50. How many panes of glass in a house lighted as follows? In the lower story there are 12 windows, each containing 12 panes; in the second story, there are 12 windows, eight of which contain 12 panes, and 4 contain 9; and in the third story there are 12 windows, each containing 9 panes. Ans. 384 panes.

51. A and B start from the same place, and travel the same road. A goes at the rate of 96 miles a day, and B goes at the rate of 57 miles a day. How far apart will they be at the end of the 9th day? Ans. 351 miles.

DIVISION.

DEFINITIONS AND RULES.

DIVISION is the operation of finding how many times one number can be taken from another.

The *dividend* is the number which is to be divided.

The *divisor* is the number by which the dividend is to be divided.

The *quotient* is the *answer*, or the number which shows how many times the divisor is contained in the dividend.

The *remainder* is the number that would be left, after taking the divisor from the dividend as many times as possible.

There are two operations in division, which are called *Short Division*, and *Long Division*. Short Division is used when the divisor is *not* more than 12. Long Division is used when the divisor *is* more than 12.

RULE FOR SHORT DIVISION.

First, State the question by putting the divisor on the left of the dividend, from which it must be separated by a curved line; and then draw a line under the dividend.

Second, See how many of the left hand figures of the dividend, it will take to make a number as large as the divisor. Find how many times the divisor is contained in that number, and set the result directly underneath.

What is division? What is the dividend? What is the divisor? What is the quotient? What is the remainder? How many kinds of operation in division. What division when the divisor is not more than 12? What division when the divisor is more than 12? What is the first step in short division? What is the second?

Third, If there be any remainder, suppose it to be prefixed to the next figure in the dividend; and, with a number thus made, find the next quotient figure.

Fourth, Proceed in this manner, making a quotient figure for each of the remaining figures in the dividend. But if at any time the figure in the dividend is less than the divisor, set down 0 in the quotient, and carry as before.

If the divisor is 10, 100, 1000, *or any number made up of* 1 *and ciphers*, cut off from the right hand of the dividend as many figures as there are ciphers in the divisor. The figures *not* cut off will be the answer, and those that are cut off, will be the remainder.

For Federal Money.—If the dividend consists of dollars only, suppose *two* ciphers annexed to it for cents, and *one* for mills; and to them, the remainder of dollars may be carried as above.

Rule for Long Division.

First, State the question by drawing a curved line on the left of the dividend, and another curve on the right; and placing the divisor on the left.

Second, Find the first partial dividend, by counting off as many of the left hand figures of the dividend as there are figures in the divisor.

But, if this collection, when taken alone, is not as great as the divisor, make it one figure longer.

Third, Find how many times this partial dividend contains the divisor, and place the number at the right of the dividend for the first figure of the quotient.

Fourth, Multiply the divisor by the figure just put in the quotient, and place the product under the first partial dividend.

Fifth, Subtract that product from the partial dividend, and to the right of the remainder, bring down the next

---

What is the third? Fourth? In performing long division, what is the first step? Second? Third? Fourth? Fifth?

figure of the dividend. And the remainder thus increased, becomes the next partial dividend.

Sixth, Find how many times this partial dividend, will contain the divisor, place the number of times to the right of the last quotient figure, and proceed as before.

Seventh, Continue this series of operations until there are no more figures in the dividend to be brought down; taking care, if any partial dividend be smaller than the divisor, to make a cipher for the quotient figure, and bring down one more figure from the dividend.

NOTE.—The following is a good method for finding how many times the divisor is contained in a partial dividend.

First find a *supposed* quotient, by seeing how many times the first figure of the divisor is contained in the first figure of the partial dividend; or, if it is not contained in that figure, see how many times it is contained in the first two figures. Then multiply mentally the first figure of the divisor by this supposed quotient, add in what would be carried if you had multiplied the second figure, and see how the sum will correspond with the number represented by so much of the partial dividend as was the figure or figures that were at first divided. If it is not greater, the supposed quotient is probably the true one; but if it is greater, make the supposed quotient one less, and try again.

But notwithstanding all this care, it will sometimes happen that when we suppose we have found the *true* quotient figure, we shall be mistaken. In such cases, it can be rectified after the product of the quotient figure into the whole divisor, is put under the partial dividend. For if the product is greater than the partial dividend, the quotient figure is too large. And after the product has been subtracted from the partial dividend, if the remainder

What is the sixth? Seventh? What is a good method for finding the true quotient figure? Is this rule always certain? How may a mistake be rectified? What other assistance may be obtained for finding the quotient figures?

is greater than the divisor, it may be known that the quotient figure is too small. After having found a few quotient figures, the pupil may be assisted in finding others by comparing his partial dividend with some of the preceding ones.

PROOF.—Multiply the quotient by the divisor; and, to the product, add the remainder, if there be any. The product will be the same as the dividend.

APPLICATION OF THE RULES.

1. How many pounds of sugar at 8 cents a pound can be bought for 488 cents?

Ans. *As many pounds as there are collections of* 8 *cents in* 488 *cents. The answer is* 61 *pounds.*

$$8\overline{)488}\quad 61$$

2. How many cows at 15 dollars a piece may be purchased for 225 dollars?

Ans. *As many cows, as there are collections of* 15 *dollars contained in* 225 *dollars. The answer is,* 15 *cows.*

$$\begin{array}{r|l} 15 & 225 \\ \hline & 15 \\ \hline & 75 \\ & 75 \\ \hline & 0 \end{array}$$

☞ In the same method, the pupil may answer all the following sums, before he does them on his slate, or when he recites in class.

3. How many hogsheads of sugar worth 37 dollars a piece can be bought for 1591 dollars? Ans. 43 hogsheads.

4. How many pieces of cloth worth 94 dollars a piece can be bought for 9964 dollars? Ans. 106 pieces.

5. How long will it take a man to travel 288 miles, if he goes 48 miles a day? Ans. 6 days.

How is division proved?

6. At 17 dollars a barrel, how many barrels of pork may be purchased for 1428 dollars? Ans. 84 barrels.

7. At 8 dollars a ton, how many tons of coal will cost 7256 dollars? Ans. 907 tons.

8. From Baltimore to Philadelphia, is 99 miles; and from Philadelphia to New York, is 90 miles. How long will it take to travel from Baltimore to New York, if we go 63 miles a day? Ans. 3 days.

9. A merchant has received 7865 bushels of salt, which must be boated to the wharf. How many boat loads will there be, if a boat can carry 65 bushels? Ans. 121 loads.

10. How many years are there in 12775 days allowing 365 days to a year? Ans. 35 years.

11. How many years are there in 17890 days?
Ans. 49 years and 5 days.

12. How many years are there in 29960 days?
Ans. 82 years and 30 days.

13. How many days are there in 1488 hours?
Ans. 62 days.

14. How many days are there in 7280 hours?
Ans. 303 days and 8 hours.

15. How many days are there in 8760 hours; and how many years will that answer make?
Ans. 365 days, which make 1 year.

16. How many hours are there in 1440 minutes?
Ans. 24 hours.

17. How many hours are there in 2700 minutes?
Ans. 45 hours.

18. How many minutes in 3600 seconds?
Ans. 60 minutes.

19. How many minutes are there in 5445 seconds?
Ans. 90 minutes and 45 seconds.

20. If 4 gills make 1 pint, how many pints are there in 36 gills? Ans. 9 pints.

21. How many pints are there in 96 gills?
Ans. 24 pints.

Section II.

1. How many pints are there in 288 gills? Ans. 72.

2. Two pints make a quart; how many quarts are there in 8 pints? Ans. 4 quarts.

3. How many quarts are in 36 pints? Ans. 18 quarts.

4. How many quarts are in 354 pints? Ans. 177 pints.

5. Four quarts make a gallon; how many gallons are there in 16 quarts? Ans. 4 gallons.

6. How many gallons are in 72 quarts? Ans. 18 gals.

7. How many gallons are in 672 quarts? Ans. 168 gals.

8. Suppose a volume to contain 2344125 letters, of which there are 4465 on a page; how many pages are there? 525.

9. A jeweler sold a quantity of silver spoons at 8 dollars a dozen, and received for them $952. How many dozen did he sell? Ans. 119 dozen.

10. If a laborer charges you $12 a month, for how many months can you hire him with $168. Ans. 14 months.

11. A farmer spent $3682 in western land, at $7 an acre. How many acres did he buy? Ans. 526 acres.

12. Supposing a field to contain 168750 hills of corn, equally divided into rows of 375 hills each. How many rows are there in the field? Ans. 450 rows.

13. A farmer sold a quantity of cheese for 2996 cents, at 7 cents a pound. How many pounds did he sell? 428.

14. If a man can travel 24 miles a day, how many days will it take him to travel 288 miles? Ans. 12 days.

15. How many acres of land, at $22 an acre, can be bought for $8514? Ans. 387 acres.

16. If a man earns $36 a month, how many months will it take him to earn $540? Ans. 15 months.

17. John said he was 12045 days old. Supposing the year to contain 365 days, how many years old was he? Ans. 33 years.

18. A ship's company received $425 a piece, which amounted to $19125 for them all. How many men were there? Ans. 45 men.

19. If a rail car goes 37 miles a minute, how long would it be in going 35113 miles? Ans. 949 minutes.

20. A merchant spent $16223.01 in pork at $18.93 per barrel. How many barrels did he get? Ans. 857 barrels.

21. Divide 218875 by 425. Ans. 515.

22. Divide 309136 by 556. Ans. 556.

23. Divide 2537682 by 74. Ans. 34293.

24. Divide 4280822 by 91. Ans. 47042.

25. Divide 62175 by 75. Ans. 829.

26. Divide 251104 by 472. Ans. 532.

27. Divide 1893312 by 912. Ans. 2076.

28. Divide 90258571 by 1435. Ans. 62897+1376.

29. Divide 863256 by 736. Ans. 1172+664.

30. How much wheat at $1.15 a bushel will pay for 27 bushels of salt that cost $26.45? Ans. 23 bushels.

31. How much will 49 bushels of salt come to, at 76 cents a bushel; and how many bushels of oats will pay for it, at 38 cents a bushel? Ans. 98 bushels.

32. How much tea at 80 cents a pound will pay for 560 pounds of beef, worth 5 cents a pound? Ans. 35 pounds.

33. How many pounds of coffee at 14 cents a pound will pay for 56 pounds of butter, worth 20 cents a pound? Ans. 80 pounds.

34. How many cords of wood at 8 dollars a cord will pay for 8 barrels of flour, worth 5 dollars a barrel? Ans. 5 cords.

35. How many tons of hay at 28 dollars a ton, will pay for 7 horses worth 64 dollars a piece? Ans. 16 tons.

36. A man bought 12 pounds of tea at $1.25 a pound; and 6 pounds of sugar at 12 cents a pound. How many dozen of eggs will pay for it all at 12 cents a dozen? Ans. 131 dozen.

37. A man bought 14 pounds of coffee at 15 cents a pound; and 16 pounds of sugar at 9 cents a pound; and paid 84 cents. How much butter, worth 18 cents a ponnd, will pay for the rest? Ans. 15 pounds.

38. How much sugar worth 9 cents a pound, must be given for 246 pounds of sugar worth 6 cents a pound?
Ans. 164 pounds.

39. How much tea at 64 cents a pound, must be given for 224 pounds of coffee, worth 20 cents a pound?
Ans. 70 pounds.

Abbreviation.

As the sum may be performed by *short* division, when the divisor is any number as small as 12; so it can be performed in the same manner, if the divisor is made up of a small number, with a cipher or ciphers annexed to it. If it can be remembered that 7 are contained 3 times in 21, it can as well be remembered that 7 hundred are contained 3 times in 21 hundred, or that 7 tens are contained 3 times in 21 tens or 210. On this principle we establish the following

Rule.

When the divisor consists of one significant figure and one or more ciphers, cut off the ciphers from the end of the divisor; and to compensate for this, cut off the same number of figures from the end of the dividend. Then divide by short division.

How many times 70 are there in 1470?
Ans. The question is the same as how many times 7 tens are in 147 tens. The answer is 21.

[40th sum.]

$$7,0\overline{)147,0}$$
$$21$$

41. How many times 800 in 792000? Ans. 990.

42. How many times 4000 in 67620000? Ans. 16905.

43. How many times 9000 in 476550000? Ans. 52950.

44. How many times 900 in 76548; or what is the same, how many times 9 hundred in 765 hundred and 48.

Ans. 9 hundred is contained in 765 hundred, 85 times, and the 48 remains.

45. How many times 4000 in 976431?

Ans. 244, and 431 remains.

46. How many times 6000 in 45672342?

Ans. 7612, and 342 remains.

47. How many times 800 in 47653?

Ans. 8 hundred is contained in 476 hundred, 59 times, and 4 hundred besides the 53 remains. The answer is 59, and 453 remains.

The pupil may now be told that this sign, $\div$ a horizontal line with one dot above and one below, signifies *divided by*; as $12 \div 3 = 4$; that is, 12 divided by 3 equals 4.

48. $4237564 \div 131 = 32347 + 107$.

49. $28191423 - 47655 \div 137 = 205428 + 132$ rem.

50. $73145970 \div 18765 \div 85 = 45 + 73$.

51. $4765856 - 781746 \div 172 = 23163 + 74$ rem.

52. $84396521 \div 70 = 1205664 + 41$ rem.

53. $176987235 \div 500 = 353974 + 235$ rem.

54. $90465767 \div 12000 = 7538 + 9767$ rem.

55. $89105742 \div 700 = 127293 + 642$ rem.

FRACTIONAL DIVISION.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

SECTION I.

1. If you wished to divide an apple equally between two of your companions, how could you do it?

Ans. *Cut it into two equal parts.*

2. What would each of those parts be called? Ans. *A half.*

3. If you should cut a yard of tape into three equal parts, what would each of those parts be called?

Answer. *One third.* *One third.* *One third.*

4. If a yard of riband be cut into four equal parts, what would each of those parts be called?

Answer. *One fourth.* *One fourth.* *One fourth.* *One fourth.*

5. If a yard of cloth be cut into five equal parts, what would one of those parts be called?

Answer. *One fifth.* *One fifth.* *One fifth.* *One fifth.* *One fifth.*

6. How many fifths of a yard make a whole yard?

7. How many fourths of a yard make a whole yard?

8. How many thirds of a yard make a whole yard?

9. How many halves of an apple make a whole apple?

10. If a ruler be divided into six equal parts, what would each of those parts be called?

Ans. *One sixth.*

11. How many sixths make a whole one?

12. If a ruler be divided into seven equal parts, what would each of those parts be called?

Ans. *One seventh.*

13. How many sevenths make a whole?

14. If a yard of cloth be divided into eight equal parts what would one of those parts be called?

Ans. *One eighth.*

15. How many eighths make a whole one?

16. If a piece of tape be divided into nine equal parts, what would one of those parts be called?

Ans. *One ninth.*

17. How many ninths make a whole thing?

18. What do you call the parts made by dividing a whole thing into ten equal parts? What do you call those made by dividing a thing into eleven equal parts? Into twelve? Fifteen? Twenty? Thirty-seven? Fifty-six? Seventy-five? Ninety-four? One hundred and thirty-five?

19. If you divide nine beans into three equal piles, what part of the whole 9 would be in each pile? Ans. *One third of them.*

20. How many then is one third of 9?

21. How many is one third of 12? Why? (*See question* 19.) Of 18? Why? Of 27? Why? Of 15? 21? 30? 24? 36?

22. How do we find a third of any number?

Ans. *Divide by* 3.

23. How do you find a fourth of any number?

24. How many is one fourth of 12? Why? Of 16? Why? Of 28? 36? 20? 32? 40? 24? 44?

25. How do you obtain the fifth of any number?

26. What is the fifth of 10? 20? 15? 30? 25? 40? 50? 35? 45? 60?

27. What is the sixth of 6? 12? 36? 24? 48? 18? 54? 30? 42? 60? 72?

28. What is a seventh of 7? 21? 14? 42? 70? 63? 28? 49? 56? 35? 77?

29. What is an eighth of 8? 16? 32? 40? 24? 56? 48? 64? 80? 72?

30. What is the ninth of 9? 27? 18? 36? 54?, 45? 63? 81? 72? 90?

INDUCTIVE EXERCISES FOR THE SLATE.

Section I.

1. Let it be required to divide 48 dollars among 8 men.

It is evident we might begin to divide the money, by giving to each man one dollar; and then afterwards give each one dollar again; and so on till we have given the whole 48 dollars.

Now, it will take 8 dollars to give to each of them 1; therefore, we can give each of them 1 as often as there is 8 in 48. Hence we see that this kind of operation can be performed by division. But although the operation is by division, the method of enunciating the answer is a little different; as in this case, we say we will give to each man *one eighth of the* 48 *dollars.*

2. A parent wishes to divide 72 apples equally among his 6 children. How many must he give to each?

Ans. *If he gives to six children* 72 *apples, he must give to each child one sixth of* 72 *apples; which is* 12 *apples.*

$$6\overline{)72}\quad 12$$

3. A man paid 63 dollars for 9 yards of cloth; what was the price of one yard?

4. If 8 loads of hay cost 112 dollars, what is the cost of 1 load?

5. If 12 reams of paper cost 48 dollars, what will 1 ream cost?

6. If 6 yards of cloth cost 96 cents, what will be the cost of 1 yard?

7. If 5 brooms cost 145 cents, what is the cost of 1 of them?

8. If 12 inkstands cost 216 cents, what is the cost of one of them?

9. If 27 barrels of pork cost 675 dollars, how much is that per barrel. Ans. 25 dollars.

10. If a gentleman distribute $365.82 to 13 poor men; how much does he give to each? Ans. $28.14.

11. If 117 pounds of sugar cost $17.55, what is it worth a pound? Ans. 15 cents.

12. If 150 pounds of tea cost $337.50, what is the cost of one pound? Ans. $2.25.

13. A person wishes to go a journey of 3264 miles in 24 days; how many miles must he go each day? Ans. 136.

14. If 2072 trees were planted equally in 28 rows, how many would there be in each row? Ans. 74 trees.

15. A merchant wishes to pack 17484 pounds of tea in 94 boxes. How many pounds must be in each box? 186.

16. A man sold 22 oxen for $792. How much was that apiece? Ans. 36 dollars.

17. A man works 15 months for $540. How much was that a month? Ans. 36 dollars.

18. A merchant paid $252,00 for 72 yards of cloth. How much was that a yard? Ans. $3.50.

19. Bought 250 yards of cloth for $1250. How much was it a yard? Ans. $5.

20. In a field of corn containing 168750 hills, there are 450 rows. How many hills are in each row? Ans. 375.

21. In a school of 25 scholars, was distributed 3475 nuts. How many had each scholar? Ans. 139.

22. If 3950 pounds of coffee cost $1145.50; what was the cost of 1 pound? Ans. 29 cents.

23. If 16 barrels of flour cost $72.00, what is the cost of one barrel? Ans. $4.50

24. If a butcher paid $252.90 for 18 oxen, how much did he pay for each? Ans. $14.05.

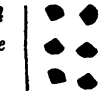
25. What is the one seventy-fifth of 8625. Ans. 115.

26. What is one thirty-eighth of 62700? Ans. 1650.

27. What is one thirty-seventh of $101.75? Ans. $2.75.

MENTAL EXERCISES.

SECTION II.



1. How can you divide 2 apples among 3 boys?

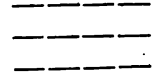
Ans. *I would first give to each boy a third of one apple, and then give to each a third of the other.*

2. How many thirds would each boy have?

3. Then what is 1 third of 2?

4. How would you divide 3 sticks of candy among 4 boys? Ans. *I would first give to each boy one fourth of one stick, and then one fourth of another, and then 1 fourth of the other.*

5. Then what is 1 *fourth* of 3?

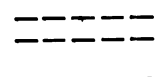


6. How would you divide 3 sticks of candy among 5 companions?

7. Then how much is 1 *fifth* of 3?

8. How would you divide 4 rolls of candy among six companions?

9. Then how much is 1 *sixth* of 4?



10. How would you divide 4 rolls of candy among 7 companions?

11. Then how much is 1 *seventh* of 4?

12. How would you divide 3 cakes among 8 companions?

13. Then what is 1 *eighth* of 3?

14. How would you divide 4 leaves into 9 equal parts?

15. Then what is 1 *ninth* of 4?

16. What is 1 fifth of 2? 1 fifth of 4?

17. What is 1 sixth of 2? 1 sixth of 3? 1 sixth of 5?

18. What is 1 seventh of 2? 1 seventh of 3? 1 seventh of 5? 1 seventh of 6?

19. What is 1 eighth of 2? 1 eighth of 4? 1 eighth of 5? 1 eighth of 6? 1 eighth of 7?

20. What is 1 ninth of 2? 1 ninth of 3? 1 ninth of 5? 1 ninth of 6? 1 ninth of 7? 1 ninth of 8?

21. What is 1 tenth of 1? 1 tenth of 2? 1 tenth of 3? 1 tenth of 4? 1 tenth of 5? 1 tenth of 6? 1 tenth of 7?

Section III.

22. What is one third of 10? Ans. *One third of* 9 *is* 3; *and one third of the* 1 *that is left, is one third. Therefore, one third of* 10 *is* 3 *and one third.*

23. What is one third of 13? 22? 17? 29? 16? 25? 14? 20? 26? 19? 28? 37?

24. What is one fourth of 17? 21? 10? 19? 14? 30? 27? 37? 41? 18? 38? 43? 15?

25. What is one fifth of 13? 34? 42? 26? 54? 48?

26. What is one sixth of 19? 32? 27? 55? 20? 33?

27. What is one seventh of 22? 58? 17? 73? 12? 49?

28. What is one eighth of 11? 51? 21? 36? 44? 78?

29. What is one ninth of 11? 37? 46? 74? 52? 25?

Inductive Exercises for the Slate.

Section II.

2. A parent wishes to divide 47 apples equally among his 6 children. How can he do it?

We answer, he must give to each child one sixth of the 47 apples; and we divide by 6 to see how much that is.

$$6\overline{)47}\quad 7+5 \text{ remains.}$$

We find that 6 is contained in 47, seven times, and that there is a remainder of 5 apples.

But these 5 apples must now be divided among the 6 children. The most obvious method of doing this, is to divide one among them by cutting it into 6 equal pieces; then divide another in the same manner; and so on, until the whole 5 are divided. Each child will have one sixth of the first apple, one sixth of the second, and so on; till, when the 5 are divided, we find he will have 5 sixths. And, although he has a sixth of each of the *five apples*, yet in common language we say that he has *five sixths of an apple*. Each child will therefore have 7 apples and five sixths of an apple.

3. A man paid 67 dollars for 9 yards of cloth. What was the price of one yard?

Ans. *If 9 yards cost 67 dollars, 1 yard costs one ninth of 67 dollars; which is 7 dollars and four ninths of a dollar.*

4. If 8 loads of hay cost 60 dollars, what is the cost of 1 load?

5. If 5 reams of paper cost 34 dollars, what will 1 ream cost?

The pupil will now perceive that he wants some method of expressing such numbers as *five sixths*, *four ninths*, *four fifths*, &c. He may accordingly be informed that such numbers are called *fractions*, because they are *parts* of a whole; and that they must be written in a different manner from whole numbers. Therefore, when we wish to write a fraction, we write first the *number of the parts*, and then write under that number the figure that shows the *name of the parts*; and draw a line between them. One half is written thus, $\frac{1}{2}$; two thirds is written thus, $\frac{2}{3}$; seven eighths thus, $\frac{7}{8}$; &c.

The following sums may now be performed on the slate.

6. If you wished to give 4 apples to 3 boys, how many would you give to each?

3)4

$1\frac{1}{3}$ Ans.

7. If 4 sheep cost 9 dollars, what is the cost of 1 sheep?

8. If 6 horses require 23 loads of hay, how much will 1 horse require?

9. If 7 tons of coal cost 50 dollars, what is the cost of 1 ton?

10. If 8 loads of hay cost 60 dollars, what is the cost of 1 load?

11. If 7 pounds of sugar cost 60 cents, what is the cost of 1 pound?

12. If 8 barrels of beef cost 75 dollars, what is the cost of 1 barrel?

13. Find the $\frac{1}{8}$ of 765.

14. Find the $\frac{1}{7}$ of 832.

15. Find the $\frac{1}{12}$ of 765.

16. Find the $\frac{1}{10}$ of 836.

17. Find the $\frac{1}{41}$ of 767.
18. Find the $\frac{1}{30}$ of 456.
19. Find the $\frac{1}{37}$ of 3561.
20. Find the $\frac{1}{51}$ of 7634.
21. Find the $\frac{1}{67}$ of 3765.
22. Find the $\frac{1}{365}$ of 5489.

Section III.

1. In the last sums, when there has been a remainder; we have divided that, by writing the divisor *under* it. We may divide any number in the same manner. Divide 12 by 37.

Ans. *I write the divisor under the* 37, *and the answer is* $\frac{12}{37}$.

2. Divide, in the same manner, 7 by 9.

3. Divide 10 by 6.
4. Divide 3 by 4.
5. Divide 6 by 8.
6. Divide 14 by 12.
7. Divide 47 by 8.
8. Divide 304 by 9.
9. Divide 97 by 46.
10. Divide 46 by 97.
11. Divide 302 by 26.
12. Divide 26 by 302.

The pupil must be informed that fractions may be considered, merely as an expression of division; and that $\frac{4}{7} = 4 \div 7$; that $\frac{47}{8} = 47 \div 8$; $\frac{97}{4} = 97 \div 4$, &c. And it is proper for him to know now, that in his future operations it will frequently be better to *express* a division, than to go through the *operation*.

13. Divide 59 by 8.

Ans. 8 *is contained in* 59, *seven times; and a remainder of* 3, *which divided by* 8, *is* $\frac{3}{8}$. *The whole answer is* $7\frac{3}{8}$.

$$8\overline{)59}\quad 7\tfrac{3}{8}$$

14. Divide 87 by 9.
15. Divide 131 by 11.
16. Divide 149 by 12.
17. Divide 196 by 15.
18. Divide 324 by 18.
19. Divide 423 by 22.
20. Divide 626 by 27.
21. Divide 329 by 31.
22. Divide 645 by 44.
23. Divide 768 by 56.
24. Divide 946 by 62.
25. Divide 849 by 78.
26. Divide 3126 by 94.
27. Divide 2164 by 86.

28. How many times is 8 contained in 4678?
29. How many times is 9 contained in 31651?
30. How many times is 90 contained in 47601?
31. How many times is 1200 contained in 262738?
32. How many times is 7000 contained in 6765421?
33. How many times is 60000 contained in 8675928?

DEFINITIONS.

1. ARITHMETICAL figures are of two kinds; those which stand for *whole things*, and those which stand for *parts of things*.

2. Figures that stand for whole things are called *integers*, or *whole numbers*.

3. Figures that stand for parts of things are called *fractions*.

4. We form a fraction when we divide an integer into *several equal parts*, and take *one or more of them*.

5. On this account, when we write a fraction, it is necessary to use two numbers, or *terms*, which we write one above the other.

6. The *lower term* is called the *denominator*, because it gives the name. It shows into how many parts the integer has been divided.

7. The *upper term* is called the *numerator*, because it tells the number of parts that are taken.

8. Sometimes the upper term or numerator expresses *that* integer, which is supposed to be divided. Thus, if 7 is divided into 8 equal parts, one of those parts is the $\frac{1}{8}$ of 7, which is written $\frac{7}{8}$.

From this, it is evident that the eighth part of 7, and the

How many kinds of arithmetical figures are there? What are integers or whole numbers? What are fractions? How is a fraction written? What are called the terms of a fraction? Which term is the denominator; and what does it show? Which term is the numerator; and what does it show? What *two* expressions may the same fraction stand for?

seven eighths of 1 are, in effect, the same, and are written in the same manner.

9. Fractions written in this manner, are called *vulgar fractions*, because this is the *general* method of writing them.

10. *Vulgar fractions* are either proper, improper, mixed, simple, compound, or complex.

11. A *proper fraction* is smaller than a unit. And therefore its numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{5}{6}$.

12. An *improper fraction* is either equal to, or greater than a unit; and its numerator is not less than its denominator; as, $\frac{4}{4}$, $\frac{17}{9}$.

13. A *mixed number* is an integer and a proper fraction combined; as, $4\frac{3}{7}$, $7\frac{5}{8}$.

14. *Simple fractions* are parts of integers; but *compound fractions* are fractions of fractions. $\frac{3}{4}$ is a simple fraction; $\frac{2}{3}$ of $\frac{3}{5}$ is a compound fraction.

15. A *complex fraction* is that which has a fraction in one or both of its terms; as, $\frac{\frac{2}{5}}{8}$, $\frac{7}{4\frac{1}{2}}$, $\frac{4\frac{1}{2}}{3\frac{5}{7}}$.

Rules.

1. When a quantity is to be divided into any number of equal parts, and *one* of those parts is to be found; make the given quantity the dividend, and the given number of parts the divisor, and proceed as in division.

2. When a dividend or the last partial dividend is smaller than the divisor, the quotient is expressed by writing the divisor underneath the dividend, in the manner of a fraction.

What are vulgar fractions? What is a proper fraction? What is an improper fraction? What is a mixed number? What are simple fractions? What are complex fractions? How may any number be divided into parts? How is division performed when the divisor is greater than the dividend?

APPLICATION OF THE RULE.

1. If 964 dollars will pay for 15 acres of land, what will pay for 1 of the same acres?

Ans. $\frac{1}{15}$ of 964 acres, which is $64\frac{4}{15}$ acres.

$$15)964(64\frac{4}{15}$$
$$90$$
$$64$$
$$60$$
$$4$$

2. A man gave 2465 dollars for 300 acres of western land. What did he give per acre? Ans. $8\frac{65}{300}$ dollars.

3. A gentleman having died, left $74450 to his seven sons. What was the fortune of each? Ans. $10635\frac{5}{7}$ dollars.

4. A merchant has filled 18 hogsheads equally with 1126 gallons of molasses. How many gallons are there in each hogshead? Ans. $62\frac{10}{18}$ gallons.

5. If 36 pieces of cloth cost 946 dollars, what is the cost of 1 piece? Ans. $26\frac{10}{36}$ dollars.

6. If 17 pieces of cloth cost 2437 dollars, what is the cost of one piece? Ans. $143\frac{6}{17}$ dollars.

7. The President of the United States has 25000 dollars a year. How much is that a day? Ans. $68\frac{180}{365}$ dollars.

8. What is the one hundred and fifty-sixth of 6555? Ans. $42\frac{3}{156}$.

9. If a ship sails 2176 miles in 14 days, at what average rate does she sail a day? Ans. $155\frac{6}{14}$ miles.

10. The circumference of the earth is supposed to be 24911 miles. As geographers have divided the circumference into 360 degrees, how many miles are there in a degree? Ans. $69\frac{71}{300}$ miles

11. What is one twenty-fifth of 7643? Ans. $305\frac{18}{25}$.
12. What is $\frac{1}{60}$ of 970 hours? Ans. $16\frac{10}{60}$ hours.
13. What is $\frac{1}{65}$ of $4832? Ans. $74\frac{22}{65}$.
14. What is $\frac{1}{81}$ of $473? Ans. $5\frac{68}{81}$.
15. What is $\frac{1}{875}$ of 9742? Ans. $11\frac{139}{875}$.
16. What is $\frac{1}{371}$ of 74638105? Ans. $2017246\frac{3}{371}$.

17. What is the quotient of 7014596 divided by 72 ?
Ans. $97424\frac{68}{72}$.

18. A man received $7.50 for a week's labor; how much was that a day? Ans. $1.25.

19. If one acre of land costs $72, how many acres will $9598 buy? [Ans. like ques. 1, p. 87.] Ans. $133\frac{22}{72}$ acres.

20. How many hogsheads will hold 7490 gallons, allowing each hogshead to hold 67 gallons?
Ans. $111\frac{53}{67}$ hogsheads.

21. If one yard of cloth costs $5.37, how many yards may be bought for $49.50? Ans. $9\frac{117}{537}$ yards.

22. If you wished to divide 41 quarts of beer among 2 persons, how much would you give to each? and if there is a fraction of a quart, tell what that fraction is called.

Ans. $20\frac{1}{2}$ quarts. Instead of $\frac{1}{2}$ quart, the pupil may tell what that $\frac{1}{2}$ quart is called. The following questions may be performed in the same way.

23. If you wished to divide 97 gallons of water into 4 casks, how much would you put in one cask, giving the proper name to the parts of a gallon? Ans. $24\frac{1}{4}$ gallons.

24. If you wish to divide 47 pints into four casks, how much would you put in each? Ans. $11\frac{3}{4}$ pints.

25. What do you call the 24th part of a day?

26. What is the one 24th of 49 days?
Ans. 2 days and $\frac{1}{24}$ of a day, or 1 hour.

27. What is the 24th of 475 days, writing the answer as in the last question? Ans. $19\frac{19}{24}$ days.

28. Divide 876 days into 24 equal parts, writing the answer as before. Ans. $36\frac{12}{24}$ days.

29. Divide 76 hours into 60 equal parts, giving the fraction its proper name. Ans. $1\frac{16}{60}$ hours.

30. Divide 4760 hours into 60 equal parts, writing the answer as before. Ans. $79\frac{30}{60}$ hours.

31. Divide 467 minutes into 60 equal parts, giving the fraction its proper name. Ans. $7\frac{47}{60}$ minutes.

Case II.

To reduce a whole or a mixed number to an improper fraction.

1. How many 6ths of an apple are there in 1 apple?

2. Then how many 6ths of an apple are there in 2 apples? Ans. *In* 1 *apple there are* 6 *sixths; and in* 2 *apples there are two times* 6 *sixths; which is* 12 *sixths.*

3. How many 6ths are there in 3 apples? In 4? In 17?

4. How many 14ths are there in 2? In 8? In 21?

5. How many 25ths are there in 4? In 7? In 36?

6. How many 14ths are there in $2\frac{5}{14}$.

Ans. *In* 2, *there are* 2 *times* $\frac{14}{14}$, *which are* $\frac{28}{14}$; *and the* $\frac{5}{14}$ *added with them make* $\frac{33}{14}$.

$$\begin{array}{r} 2\frac{5}{14} \\ 14 \\ \hline 28 \\ 5 \\ \hline 33 = \frac{33}{14}. \end{array}$$

7. What is the improper fraction of $3\frac{5}{12}$? Ans. $\frac{41}{12}$.

8. What is the improper fraction of $4\frac{8}{13}$? Ans. $\frac{60}{13}$.

9. Reduce $5\frac{6}{11}$ to an improper fraction. Ans. $\frac{61}{11}$.

Rule.

To reduce a whole or a mixed number to an improper fraction, multiply the whole number by the required denominator; and, if it is a mixed number, add the *numerator* of the fraction; then write the result over the required denominator.

10. Reduce $7\frac{4}{15}$ to an improper fraction. Ans. $\frac{109}{15}$.

11. Reduce $74\frac{9}{22}$ to an improper fraction. Ans. $\frac{1637}{22}$.

12. Reduce $127\frac{2}{81}$ to an improper fraction. Ans. $\frac{10289}{81}$.

13. Reduce $476\frac{19}{45}$ to an improper fraction. Ans. $\frac{21439}{45}$.

14. Reduce 435 to 18ths. Ans. $\frac{7830}{18}$.

15. Reduce $643\frac{4}{17}$ to an improper fraction. Ans. $\frac{10935}{17}$.

16. In $147\frac{5}{87}$, how many 87ths? Ans. $\frac{12794}{87}$.

17. In $327\frac{19}{25}$ of a hogshead, how many $\frac{1}{25}$ of a hogshead? Ans. $\frac{8194}{25}$.

Case III.

To reduce an improper fraction to a whole or mixed number.

Rule.—Divide the numerator by the denominator.

1. In $\frac{97}{4}$, how many whole ones? *There are as many whole ones as there are collections of* 4 *fourths in* 97 *fourths; which is* $24\frac{1}{4}$.

$$4\overline{)97}\quad 24\tfrac{1}{4}$$

2. How many whole ones in $\frac{94}{13}$? Ans. $7\frac{3}{13}$.
3. Change $\frac{62}{16}$ to a whole number. Ans. $3\frac{5}{16}$.
4. Change $\frac{364}{72}$ of a dollar to dollars. Ans. $5\frac{4}{72}$.
5. Change $\frac{973}{43}$ of a pound to pounds. Ans. $22\frac{27}{43}$.
6. Change $\frac{4765}{37}$ of a year to years. Ans. $128\frac{29}{37}$.
7. Change $\frac{8746}{87}$ to a whole number. Ans. $100\frac{46}{87}$.
8. Reduce $\frac{1025}{23}$ to a whole number. Ans. $44\frac{13}{23}$.
9. Reduce $\frac{7318}{35}$ to a whole number. Ans. $206\frac{8}{35}$.
10. Reduce $\frac{8229}{16}$ to a whole number. Ans. $514\frac{5}{16}$.
11. Reduce $\frac{74672}{44}$ to a whole number. Ans. $1697\frac{4}{44}$.
12. Reduce $\frac{3415623}{96}$ to a whole number. Ans. $35572\frac{0}{96}$.
13. Reduce $\frac{108}{13}$ to a whole number. Ans. $8\frac{4}{13}$.
14. Reduce $\frac{307}{14}$ to a whole number. Ans. $21\frac{13}{14}$.
15. Change $\frac{362}{21}$ to a whole number. Ans. $17\frac{5}{21}$.
16. Change $\frac{14271}{38}$ to a whole number. Ans. $375\frac{21}{38}$.
17. Change $\frac{2765}{25}$ to a whole number. Ans. $110\frac{15}{25}$.
18. Change $\frac{426}{21}$ of a pound to pounds. Ans. $20\frac{6}{21}$ lbs.
19. Change $\$\frac{3271}{18}$ to dollars. Ans. $\$181\frac{3}{18}$.
20. Change $\frac{1341}{42}$ of a minute to minutes. Ans. $31\frac{39}{42}$ m.
21. Change $\frac{37665}{141}$ of a mile to miles. Ans. $266\frac{99}{141}$ m.
22. Change $\frac{2786}{12}$ of a rod to rods. Ans. $385\frac{6}{12}$ rods.
23. Change $\frac{12364}{103}$ to a whole number. Ans. $120\frac{28}{103}$.
24. Change $\frac{78655}{276}$ to a whole number. Ans. $285\frac{137}{276}$.

ADDITION OF FRACTIONS.

1. A man gave some money to his four children as follows. To one he gave $\frac{7}{8}$ of a dollar; to another, $\frac{5}{8}$ of a dollar; to another, $\frac{3}{8}$ of a dollar; and to another, $\frac{1}{8}$ of a dollar. How many eighths of a dollar did he give to them all? And how many dollars will that make?

2. In a certain field, $\frac{3}{10}$ of an acre bears wheat; $\frac{4}{10}$ of an acre bears rye; $\frac{9}{10}$ of an acre bears corn; $\frac{7}{10}$ of an acre bears oats; and $\frac{6}{10}$ of an acre bears barley. How many tenths of an acre are there in the whole field? And how many acres does that equal?

Rule.

Fractions of the same denominator are added, by adding only the numerators; and considering the denominator as merely the name.

3. Add together $\frac{3}{11}$, $\frac{4}{11}$, $\frac{5}{11}$, $\frac{6}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, and $\frac{9}{11}$. And tell how many whole ones in the amount.

Ans. $\frac{42}{11}$; *which equals* $3\frac{9}{11}$. | $11)42$, $3\frac{9}{11}$

4. How much is $\frac{4}{21}$, $\frac{10}{21}$, $\frac{11}{21}$, $\frac{16}{21}$, $\frac{18}{21}$, and $\frac{10}{21}$? Ans. $3\frac{15}{21}$.

5. How much is $\frac{7}{32}$, $\frac{26}{32}$, $\frac{35}{32}$, $\frac{14}{32}$, $\frac{11}{32}$, and $\frac{19}{32}$? Ans. $3\frac{16}{32}$.

6. How much is $\frac{27}{47}$, $\frac{42}{47}$, $\frac{5}{47}$, $\frac{21}{47}$, $\frac{39}{47}$, and $\frac{18}{47}$? Ans. $3\frac{11}{47}$.

7. How much is $\frac{18}{25}$, $\frac{2}{25}$, $\frac{17}{25}$, $\frac{24}{25}$, $\frac{16}{25}$, and $\frac{11}{25}$? Ans. $3\frac{13}{25}$.

8. How much is $\frac{39}{64}$, $\frac{16}{64}$, $\frac{26}{64}$, $\frac{35}{64}$, $\frac{18}{64}$, and $\frac{63}{64}$? Ans. $3\frac{5}{64}$.

9. How much is $\frac{47}{72}$, $\frac{19}{72}$, $\frac{48}{72}$, $\frac{57}{72}$, $\frac{62}{72}$, and $\frac{32}{72}$? Ans. $3\frac{8}{72}$.

10. How much is $\frac{81}{97}$, $\frac{46}{97}$, $\frac{89}{97}$, $\frac{76}{97}$, $\frac{85}{97}$, and $\frac{92}{97}$? Ans. $4\frac{81}{97}$.

11. How much is $\frac{47}{64}$, $\frac{39}{64}$, $\frac{62}{64}$, $\frac{29}{64}$, $\frac{42}{64}$, and $\frac{51}{64}$? Ans. $4\frac{14}{64}$.

12. How much is $\frac{62}{47}$, $\frac{39}{47}$, $\frac{2}{47}$, $\frac{94}{47}$, $\frac{161}{47}$, and $\frac{71}{47}$? Ans. $9\frac{8}{47}$.

13. How much is $\frac{64}{50}$, $\frac{4}{50}$, $\frac{27}{50}$, $\frac{96}{50}$, $\frac{17}{50}$, and $\frac{133}{50}$? Ans. $5\frac{35}{50}$.

14. How much is $\frac{111}{126}$, $\frac{76}{126}$, $\frac{44}{126}$, $\frac{79}{126}$, and $\frac{142}{126}$? Ans. $\frac{474}{126}$.

15. How much is $\frac{10}{107}$, $\frac{46}{107}$, $\frac{102}{107}$, $\frac{91}{107}$, and $\frac{17}{107}$? Ans. $2\frac{43}{107}$.

Section II.

16. How much is $14\frac{3}{5}$ and $18\frac{4}{5}$?

Ans. *The fractions added together amount to* $\frac{7}{5}$ *which equals* $1\frac{2}{5}$. *The whole numbers with the* 1 *added in equals* 33. *The answer is* $34\frac{2}{5}$.

Numerators of the fractions.

$14\frac{3}{5}$ 3
$18\frac{4}{5}$ 4
5)7
$33\frac{2}{5}$ $1\frac{2}{5}$

17. How much is $3\frac{7}{12}$ and $5\frac{11}{12}$? Ans. $9\frac{6}{12}$.

18. How much is $\frac{1}{12}$, $\frac{3}{12}$, $4\frac{2}{12}$, $3\frac{5}{12}$, and $1\frac{7}{12}$? Ans. $9\frac{6}{12}$.

19. How much is $\frac{6}{14}$, $2\frac{3}{14}$, $\frac{5}{14}$, $1\frac{7}{14}$, and $3\frac{13}{14}$? Ans. $8\frac{5}{14}$.

20. How much is $1\frac{7}{12}$, $2\frac{10}{12}$, $5\frac{8}{12}$, $7\frac{9}{12}$, and $\frac{3}{12}$? Ans. $18\frac{1}{12}$.

21. Add 7, $8\frac{45}{67}$, $14\frac{2}{67}$, $9\frac{4}{67}$, $\frac{59}{67}$, and $101\frac{42}{67}$? Ans. $141\frac{18}{67}$.

22. Add $37\frac{48}{107}$, $15\frac{93}{107}$, $\frac{101}{107}$, and $171\frac{45}{107}$. Ans. $225\frac{73}{107}$.

23. Add together $30\frac{19}{25}$, $46\frac{23}{25}$, $9\frac{6}{25}$, and $16\frac{12}{25}$. Ans. $103\frac{10}{25}$.

24. Add $27\frac{34}{40}$, $102\frac{27}{40}$, $66\frac{2}{40}$, and $41\frac{11}{40}$. Ans. $238\frac{6}{40}$.

25. Add together $37\frac{41}{83}$, $29\frac{17}{83}$, $46\frac{63}{83}$, and $81\frac{72}{83}$. Ans. $195\frac{27}{83}$.

26. Add together 42, $\frac{67}{84}$, $27\frac{46}{84}$, 16, and $39\frac{79}{84}$. Ans. $126\frac{24}{84}$.

SUBTRACTION OF FRACTIONS.

1. A father gave to his eldest son, $\frac{5}{8}$ of an orange; and to his youngest son, $\frac{3}{8}$ of an orange. How much did he give to the oldest more than to the youngest?

2. How much is $\frac{9}{14}$ of a dollar more than $\frac{3}{14}$ of a dollar?

3. How much is $\frac{17}{20}$ more than $\frac{6}{20}$?

Rule.

Subtract the numerator of the smallest number from the numerator of the larger number. When a whole number precedes the fraction, 1 may be borrowed from it if necessary.

4. Subtract $\frac{14}{37}$ from $\frac{32}{37}$.

$\frac{32}{14}$ Ans. $\frac{18}{37}$.
$\frac{18}{}$

5. Subtract $\frac{27}{84}$ from $\frac{43}{84}$. Ans. $\frac{16}{84}$.

6. Subtract $\frac{39}{101}$ from $\frac{92}{101}$. Ans. $\frac{53}{101}$.

7. Subtract $\frac{41}{147}$ from $\frac{116}{147}$. Ans. $\frac{75}{147}$.

8. Subtract $\frac{19}{129}$ from $\frac{123}{129}$. Ans. $\frac{104}{129}$.

9. Subtract $\frac{12}{16}$ from 1.

Ans. 1 *equals* $\frac{16}{16}$; *and* $\frac{12}{16}$ *from* $\frac{16}{16}$, *leaves* $\frac{4}{16}$.

$$\begin{array}{r|l} 1 & \\ & \frac{12}{16} \\ & \frac{4}{16} \end{array}$$

10. Subtract $\frac{9}{41}$ from 1. Ans. $\frac{32}{41}$.

11. Subtract $\frac{7}{36}$ from 1. Ans. $\frac{29}{36}$.

Section II.

12. Subtract $6\frac{6}{12}$ from $9\frac{4}{12}$. Ans. $\frac{6}{12}$

from $\frac{4}{12}$, *I cannot. I borrow* 1 *from* 9, *which equals* $\frac{12}{12}$; *and* $\frac{4}{12}$ *added with it, makes* $\frac{16}{12}$. $\frac{6}{12}$ *from* $\frac{16}{12}$ *leaves* $\frac{10}{12}$. 6 *from* 8, *leaves* 2. *The answer is* $2\frac{10}{12}$.

	Numerators.
$9\frac{4}{12}$	12 4
$6\frac{6}{12}$	16 6
$2\frac{10}{12}$	10

13. Subtract $17\frac{4}{15}$ from $25\frac{3}{15}$. Ans. $7\frac{14}{15}$.

14. Subtract $12\frac{5}{21}$ from $17\frac{2}{21}$. Ans. $4\frac{18}{21}$.

15. Subtract $41\frac{6}{29}$ from $87\frac{5}{29}$. Ans. $45\frac{28}{29}$.

16. Subtract $28\frac{12}{33}$ from $49\frac{8}{33}$. Ans. $20\frac{29}{33}$.

17. Subtract $37\frac{22}{41}$ from $56\frac{12}{41}$. Ans. $18\frac{31}{41}$.

18. Subtract $45\frac{76}{105}$ from $54\frac{67}{105}$. Ans. $8\frac{96}{105}$.

19. Subtract $676\frac{445}{737}$ from $895\frac{327}{737}$. Ans. $218\frac{839}{737}$.

20. Subtract $45\frac{403}{876}$ from $472\frac{327}{876}$. Ans. $426\frac{710}{876}$.

21. Subtract $967\frac{438}{536}$ from $1211\frac{276}{536}$. Ans. $243\frac{374}{536}$.

22. Subtract $4\frac{39}{45}$ from 67. Ans, $62\frac{6}{45}$.

23. Subtract $9\frac{47}{55}$ from $45\frac{7}{55}$. Ans. $35\frac{15}{55}$.

24. Subtract $47\frac{362}{626}$ from $365\frac{65}{626}$. Ans. $317\frac{329}{626}$.

25. Subtract $12\frac{21}{35}$ from $14\frac{15}{35}$. Ans. $1\frac{29}{35}$.

26 Subtract $146\frac{308}{1008}$ from $327\frac{17}{1008}$. Ans. $180\frac{697}{1008}$.

MULTIPLICATION OF FRACTIONS.

1. WHAT is the cost of 3 yards of cloth, at $\frac{3}{4}$ of a dollar a yard? Ans. 3 *times* $\frac{3}{4}$ *of a dollar, which is* $\frac{9}{4}$ *of a dollar; which equals* $2\frac{1}{4}$ *dollars.*

2. If one pound of tea costs $\frac{14}{23}$ of a dollar, what is the cost of 8 lbs.? Ans. $\frac{112}{23} = 4\frac{20}{23}$.

3. If one horse consumes $\frac{6}{11}$ of a bushel of oats in a day, how much will 14 horses consume? Ans. $7\frac{7}{11}$ of a bush.

4. At $\frac{25}{97}$ of a dollar a pound for coffee, what would be the cost of 35 lbs.? Ans. $9\frac{2}{97}$ dollars.

5. How much is 17 times $\frac{45}{72}$? Ans. $10\frac{45}{72}$.

6. How much is 56 times $\frac{38}{121}$? Ans. $17\frac{71}{121}$.

7. Multiply $\frac{142}{329}$ by 67. Ans. $28\frac{303}{329}$.

8. If the interest on 1 dollar for a year is $\frac{6}{100}$ of a dollar, what is the interest on 414 dollars, for the same time? Ans. $24\frac{84}{100}$ dollars.

9. Multiply $\frac{49}{63}$ by 19. Ans. $14\frac{49}{63}$.

10. How much is 68 times $\frac{51}{63}$ of a hogshead? Ans. $55\frac{3}{63}$.

11. A lady gave to 3 children, each 2 oranges and three fourths. How many oranges did it take?

Ans. *Three times two oranges and 3 fourths. Three times* $\frac{3}{4}$ *of an orange is* $\frac{9}{4}$ *of an orange, which equals* $2\frac{1}{4}$ *oranges. Three times 2 oranges is 6 oranges; which, with the* $2\frac{1}{4}$ *oranges added in, equals* $8\frac{1}{4}$ *oranges.*

$$\begin{array}{r} 2\frac{3}{4} \\ 3 \\ \hline 2\frac{1}{4} \\ 6 \\ \hline 8\frac{1}{4} \end{array} \qquad \begin{array}{r} 3 \\ 3 \\ \hline 4)9 \\ \hline 2\frac{1}{4} \end{array}$$

12. If one ton of hay is worth \$$14\frac{7}{8}$, what is the worth of 18 tons? Ans. \$$267\frac{6}{8}$.

13. Supposing a barrel to contain $31\frac{1}{2}$ gallons; what will 12 barrels contain? Ans. 378 gals.

14. How much is 40 times $16\frac{1}{2}$ feet? Ans. 660 ft.

15. How much is 25 times $37\frac{1}{2}$ cents? Ans. \$$9.37\frac{1}{2}$.

16. If one barrel of fish is worth $9\frac{11}{12}$, what is the worth of 18 barrels? Ans. $178\frac{6}{12}$.

17. Multiply $7\frac{9}{47}$ by 8. Ans. $57\frac{25}{47}$.

18. Multiply $14\frac{4}{16}$ by 9. Ans. 128.

19. Multiply $36\frac{18}{47}$ by 4. Ans. $145\frac{25}{47}$.

20. Multiply $47\frac{127}{136}$ by 7. Ans. $335\frac{73}{136}$.

21. Multiply $29\frac{901}{1036}$ by 6. Ans. $179\frac{236}{1036}$.

22. Multiply $86\frac{43}{344}$ by 8. Ans. $689\frac{2}{344}$.

23. Multiply $327\frac{479}{875}$ by 9. Ans. $2944\frac{1036}{875}$.

24. How many apples will it take to give to 19 boys $\frac{3}{4}$ of an apple apiece? Ans. $14\frac{1}{4}$ apples

25. If a horse eats $\frac{4}{15}$ of a bushel of oats in one day, how much will he eat in 32 days? Ans. $9\frac{11}{15}$ bushels.

26. If a man who is in debt, can pay but $\frac{42}{76}$ of a dollar for each dollar that he owes, how much will he pay to a man whom he owes $336? Ans. $185\frac{52}{76}$.

27. If a man in debt can pay $\frac{314}{917}$ of a dollar for a dollar, how much will he pay to a man whom he owes $3049?

28. If a man raises $30\frac{7}{12}$ bushels of corn on 1 acre, how much does he raise at that rate on 14 acres? Ans. $428\frac{2}{12}$.

29. If a stage coach goes in 1 hour, $6\frac{18}{20}$ miles, how far will it go at that rate in 27 hours? Ans. $186\frac{6}{20}$ miles.

30. What is the cost of 87 acres of land, at $47\frac{5}{8}$ an acre? Ans. $4154\frac{3}{8}$.

31. What is the cost of 35 yards of cloth, at $\frac{7}{16}$ of a dollar a yard? Ans. $15\frac{5}{16}$.

32. If a steam boat goes $9\frac{17}{20}$ miles an hour, how far would she go in 14 hours? Ans. $137\frac{18}{20}$ miles.

33. How many bushels of corn will it take to give to 24 families $3\frac{20}{64}$ bushels a piece? Ans. $79\frac{32}{64}$.

34. What is the cost of 314 pounds of sugar at $7\frac{3}{16}$ cents per pound? Ans. $\$22.56\frac{14}{16}$.

35. What is the cost of 108 pounds of tea at $\frac{5}{8}$ of a dollar a pound? Ans. $67\frac{4}{8}$.

DIVISION OF FRACTIONS.

1. A man bought 3 brooms for $\frac{9}{10}$ of a dollar. How much was that a piece? Ans. $\frac{1}{3}$ *of* $\frac{9}{10}$, *which is* $\frac{3}{10}$.

2. A lady bought 4 yards of riband for $\frac{20}{21}$ of a dollar. How much was that a yard? Ans. $\frac{5}{21}$ of a dollar.

3. How much is $\frac{1}{9}$ of $\frac{378}{400}$? Ans. $\frac{42}{400}$.

4. How much is $\frac{1}{12}$ of $\frac{468}{572}$? Ans. $\frac{39}{572}$.

5. Divide $\frac{336}{50}$ by 14. Ans. $\frac{24}{50}$.

6. Divide $\frac{368}{41}$ by 23. Ans. $\frac{16}{41}$.

7. Divide $\frac{1428}{931}$ by 24. Ans. $\frac{62}{931}$.

8. What is $\frac{1}{65}$ of $\frac{7865}{234}$? Ans. $\frac{121}{234}$.

SECTION II.

1. A man bought 8 cows for $\$144\frac{8}{9}$. How much was that a piece? Ans. *The* $\frac{1}{8}$ *of* $\$144\frac{8}{9}$. $\frac{1}{8}$ *of* $\$144$ *is* $\$18$. *And* $\frac{1}{8}$ *of* $\frac{8}{9}$ *of a dollar, is* $\frac{1}{9}$ *of a dollar. Therefore the answer is* $\$18\frac{1}{9}$.

2. When 4 oranges cost $10\frac{2}{3}$ cents, what is the cost of 1 orange? Ans. *If 4 oranges cost* $10\frac{2}{3}$ *cents, 1 orange will cost* $\frac{1}{4}$ *of* $10\frac{2}{3}$ *cents.* $\frac{1}{4}$ *of 8 cents is 2 cents. The* $2\frac{2}{3}$ *cents that remain, equals* $\frac{8}{3}$ *of a cent; and* $\frac{1}{4}$ *of* $\frac{8}{3}$ *of a cent is* $\frac{2}{3}$ *of a cent; which added to 2 cents makes* $2\frac{2}{3}$ *cents.*

$$\begin{array}{r|l} & 2\frac{2}{3} \\ & 3 \\ \hline 4)10\frac{2}{3} & 4)8 \\ 2\frac{2}{3} & 2 \end{array}$$

RULE.

Divide the whole number as in common division; taking care to put the remainder with the fraction, and then change it to an improper fraction. Divide the numerator of this improper fraction, and put the quotient with the answer of whole numbers.

3. A person gave $\$26\frac{3}{7}$ for 5 barrels of flour. How much was that a barrel? Ans. $\$5\frac{2}{7}$.

4. A field of $13\frac{3}{8}$ acres has been divided into 4 equal lots. How much is in each lot? Ans. $3\frac{3}{8}$ acres.

5. If 4 loads of wood cost $\$19\frac{1}{3}$, what is the cost of 1 load? Ans. $\$4\frac{4}{3}$.

6. Five brothers bought $31\frac{6}{9}$ pounds of beef, which they wish to divide equally. What is each one's share?

Ans. $\frac{1}{5}$ of $31\frac{6}{9} = 6$ *and* $1\frac{6}{9}$ *over, &c.* Ans. $6\frac{3}{8}$ pounds.

7. If 12 yards of cambric cost $\$42\frac{1}{7}$, what would 1 yard cost? Ans. $\$3\frac{4}{7}$.

8. If 2 yards of merino is worth $\$3\frac{66}{240}$, what is the worth of 1 yard? Ans. $\$1\frac{153}{240}$.

9. If a man travel $122\frac{180}{320}$ miles in 5 days, how far does he travel in 1 day? Ans. $24\frac{164}{320}$ miles.

10. Bought 7 yards of broadcloth for $\$39\frac{56}{112}$; how much was that per yard? Ans. $5\frac{72}{112}$.

11. Bought a keg of wine containing 10 gallons, for $\$25\frac{7500}{36000}$; how much was that a gallon? Ans. $\$2\frac{2056}{3600}$.

12. Divide $647\frac{15}{19}$ by 4. Ans. $161\frac{18}{19}$.

13. Divide $456\frac{64}{102}$ by 16. Ans. $28\frac{55}{102}$.

14. Divide $371\frac{31}{47}$ by 22. Ans. $16\frac{42}{47}$.

15. What is $\frac{1}{47}$ of $2890\frac{108}{675}$? Ans. $61\frac{339}{875}$.

16. What is $\frac{1}{72}$ of $9763\frac{584}{4000}$? Ans. $135\frac{2397}{4000}$.

17. What is the quotient of $\frac{205}{4213}$ ÷ 5? Ans. $\frac{173}{4213}$.

18. What is the quotient of $\frac{45821}{96254}$ ÷ 9? Ans. $\frac{5069}{96254}$

19. Divide $\frac{2688}{9472}$ by 42. Ans. $\frac{64}{9472}$.

20. What is $\frac{1}{89}$ of $\frac{5607}{75432}$? Ans. $\frac{63}{75432}$.

21. Divide $10310\frac{70}{88}$ by 27. Ans. $381\frac{74}{88}$.

22. Divide $14322\frac{36}{93}$ by 52. Ans. $275\frac{41}{94}$.

MULTIPLICATION BY FRACTIONS.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. WHAT is the $\frac{1}{4}$ of 8? What is the $\frac{3}{4}$ of 8? Ans. $\frac{1}{4}$ *of* 8 *is* 2; *and* $\frac{3}{4}$ *of* 8, *is* 3 *times* 2; *which is* 6.

2. What is $\frac{1}{5}$ of 10? $\frac{2}{5}$ of 10? $\frac{4}{5}$ of 10?

3. What is $\frac{1}{3}$ of 15? $\frac{2}{3}$ of 15? $\frac{5}{3}$ of 15?

4. What is $\frac{1}{6}$ of 18? $\frac{4}{6}$ of 18? $\frac{7}{6}$ of 18?

5. What is $\frac{1}{9}$ of 27? $\frac{3}{9}$ of 27? $\frac{7}{9}$ of 27?

6. What is $\frac{1}{7}$ of 35? $\frac{4}{7}$ of 35? $\frac{9}{7}$ of 35?

7. What is $\frac{1}{8}$ of 96? $\frac{5}{8}$ of 96? $\frac{12}{8}$ of 96?

8. What is $\frac{1}{10}$ of 100? $\frac{3}{10}$ of 100? $\frac{7}{10}$ of 100?

9. What is $\frac{1}{4}$ of 3? $\frac{3}{4}$ of 3? $\frac{4}{4}$ of 3?

10. What is $\frac{1}{3}$ of 3? $\frac{3}{3}$ of 3? $\frac{6}{3}$ of 3?

11. What is $\frac{2}{8}$ of 7? $\frac{5}{8}$ of 7? $\frac{4}{9}$ of 7? $\frac{7}{9}$ of 7?

12. What is $\frac{4}{7}$ of 8? $\frac{6}{7}$ of 8? $\frac{2}{5}$ of 8? $\frac{5}{5}$ of 8?

13. What is $\frac{4}{6}$ of 4? $\frac{3}{3}$ of 2? $\frac{3}{4}$ of 5? $\frac{6}{7}$ of 12?

14. What is $1\frac{1}{4}$ times 8? Ans. $1\frac{1}{4}$ *times* 8 *is the same as* $\frac{5}{4}$ *of* 8. $\frac{1}{4}$ *of* 8 *is* 2, *and* $\frac{5}{4}$ *of* 8 *is* 5 *times* 2; *which is* 10.

15. What is $1\frac{1}{3}$ times 6? $2\frac{1}{3}$ times 6? $2\frac{1}{3}$ times 9?

16. What is $1\frac{2}{5}$ times 10? $1\frac{2}{3}$ times 15? $1\frac{3}{5}$ times 10?

17. What is $1\frac{5}{6}$ times 12? $1\frac{1}{8}$ times 12? $1\frac{2}{7}$ times 17?

18. What is $1\frac{1}{7}$ times 5? $1\frac{1}{4}$ times 4? $2\frac{2}{3}$ times 2?

19. What is $1\frac{3}{4}$ times 5? $1\frac{2}{3}$ times 9? $1\frac{1}{8}$ times 10?

20. What is $\frac{2}{3}$ of 20? Ans. $\frac{1}{3}$ *of* 20 *is* $6\frac{2}{3}$; *and* $\frac{2}{3}$ *of* 20 *is* 2 *times* $6\frac{2}{3}$. 2 *times* 6 *is* 12; *and* 2 *times* $\frac{2}{3}$ *is* $\frac{4}{3}$, *which equals* $1\frac{1}{3}$; *which added to* $12 = 13\frac{1}{3}$.

21. What is $\frac{1}{8}$ of 50? $\frac{3}{8}$ of 50? $\frac{5}{8}$ of 50?
22. What is $\frac{1}{5}$ of 67? $\frac{4}{5}$ of 67? $\frac{3}{5}$ of 67?
23. What is $\frac{1}{10}$ of 76? $\frac{7}{10}$ of 76? $1\frac{3}{10}$ times 76?
24. What is $\frac{3}{4}$ of 7? $\frac{1}{4}$ of 7? $2\frac{1}{4}$ times 7?
25. What is $\frac{4}{9}$ of 9? $\frac{7}{9}$ of 9? $2\frac{1}{3}$ times 9?
26. What is $\frac{2}{5}$ of 17? $\frac{10}{9}$ of 17?
27. What is $\frac{5}{6}$ of 41? $\frac{7}{8}$ of 41? $1\frac{4}{6}$ times 41?
28. What is $\frac{3}{8}$ of 67? $\frac{5}{8}$ of 67? $1\frac{1}{8}$ times 67?
29. What is $\frac{4}{7}$ of 66? $\frac{6}{7}$ of 66? $1\frac{2}{7}$ times 66?
30. What is $\frac{2}{4}$ of 35? $\frac{7}{4}$ of 35? $1\frac{3}{4}$ times 35?

INDUCTIVE EXERCISES FOR THE SLATE.

1. If oranges are worth 40 cents a dozen, what is the worth of $\frac{1}{4}$ of a dozen? And what is the worth of $\frac{3}{4}$ of a dozen?

Ans. *If one dozen is worth* 40 *cents,* $\frac{1}{4}$ *of a dozen is worth* $\frac{1}{4}$ *of* 40 *cents, which is* 10 *cents; and* $\frac{3}{4}$ *of a dozen is worth* 3 *times* 10 *cents, which is* 30 *cents.*

$$4)\overline{40}\quad 10\quad 3\quad 30$$

2. If a boat is worth $60, what is the worth of three quarters of it? Ans. $45.

3. A man gave $6.90 for a yard of cloth. What is the worth of $\frac{2}{3}$ of it? Ans. $4.60.

4. A man gave $25 for a load of corn. What is $\frac{3}{5}$ of the load worth? Ans. $15.

5. If an acre of ground cost $375, what is $\frac{8}{15}$ of it worth? Ans. $200.

6. If a man earns $936 in a year, what are his earnings in $1\frac{7}{12}$ of a year? Ans. $546.

8. If a vessel is valued at $4750, what is $\frac{3}{19}$ of her worth? Ans. $750.

9. At $8 a hundred, what is the value of $\frac{1}{3}$ of a hun-

dred of sugar?—and $\frac{3}{5}$ of a hundred?—and $\frac{4}{5}$ of a hundred? Ans. $\$1\frac{3}{5}$.—$\$4\frac{4}{5}$.—$\$6\frac{2}{5}$.

8. If a bushel of potatoes costs 30 cents, what is the cost of $\frac{1}{7}$ of a bushel? Of $\frac{4}{7}$ of a bushel? Ans. $17\frac{1}{7}$ cents.

9. If a box of oranges costs $5.75, what is the cost of $\frac{1}{9}$ of a box?—Of $\frac{3}{9}$ of a box?—and $\frac{7}{9}$ of a box?

Ans. $\$.63\frac{8}{9}$.—$\$1.91\frac{8}{9}$.—$\$4.47\frac{2}{9}$.

10. If a pound of butter costs 27 cents, what is the cost of $\frac{1}{16}$ of a lb.?—$\frac{3}{16}$ of a lb.?—and $\frac{10}{16}$ of a lb.?

Ans. $1\frac{11}{16}$ cents—$5\frac{1}{16}$ cents—and $16\frac{14}{16}$ cents.

11. If a barrel of flour costs $7.21, what is the cost of $\frac{2}{96}$ of a barrel?—$\frac{7}{96}$ of a barrel?—$\frac{42}{96}$ of a barrel?

[First for $\frac{1}{96}$.] Ans. $\$0.15\frac{2}{96}$.—$\$0.52\frac{55}{96}$.—$\$3.15\frac{42}{96}$.

12. If a man travels 96 miles in a day, how far will he travel in $\frac{1}{2}$ of a day? How far in $\frac{3}{2}$ of a day?

Ans. 48 miles.—144 miles.

13. If a man travels 96 miles in a day, how far will he travel in $\frac{1}{24}$ of a day? How far in $5\frac{1}{2}$ days?

Ans. $5\frac{1}{2}$ days $= \frac{11}{2}$ days. He will travel 528 miles.

14. A man bought 5 sheep for $25. What was the cost of $\frac{3}{5}$ of them, or 3 sheep? Ans. $15.

15. A man bought 8 gallons of wine for $10.00. What was the cost of $\frac{5}{8}$ of them, or 5 gallons? Ans. $6.25.

16. A man traveled 60 miles in 8 hours. How far did he travel in $\frac{3}{8}$ of the time, or 3 hours? Ans. $22\frac{4}{8}$ miles.

17. If 14 yards of cloth cost $75.00, what is the cost of $\frac{9}{14}$ as much, or 9 yards? Ans. $\$48.21\frac{6}{14}$.

18. A ship that was sold for $38500.00, belonged to two men. A owned $\frac{7}{13}$ of it; and B owned $\frac{6}{13}$ of it. How much money belonged to each?

Ans. A's share $\$20730.76\frac{12}{13}$; B's, $\$17769.23\frac{1}{13}$.

19. If 4 horses consume 6 bushels of oats in a week, how many bushels will be consumed by $\frac{7}{4}$ as many, or 7 horses? Ans. $10\frac{1}{2}$ bushels

20. If a man give 123 cents for 7 yards of calico, what must he give for $\frac{2}{7}$ as much, or 9 yards? Ans. $158\frac{1}{7}$ cts.

21. If the tuition of 3 boys cost $40, what would be the tuition of 5 boys at the same rate? Ans. $66\frac{2}{3}$.

Section II.

Divide by writing the divisor underneath.

1. What is the $\frac{1}{37}$ of 72? Ans. $\frac{72}{37}$.

Rule.—It was stated on page 102, that division is frequently expressed by putting the divisor under the dividend, simply giving it the form of a fraction. This method of operating is very convenient in the following sums.

2. What is $\frac{14}{35}$ of $17.34?

Ans. $\frac{1}{35}$ *of* 1734 *cents is* $\frac{1734}{35}$ *of a cent; and* $\frac{14}{35}$ *of* 1734 *cents, is* 14 *times* $\frac{1734}{35}$ *of a cent; which equals* $\frac{24276}{35}$ *of a cent; which* $= 693\frac{21}{35}$ *cents or,* $6.93\frac{21}{35}$.

$$\begin{array}{r} 1734 \\ \times\ 14 \\ \hline 6936 \\ 1734 \\ \hline 35\overline{)24276} \end{array}$$

3. If 74 pounds of sugar cost $5, what is the cost of 27 pounds? Ans. *If* 74 *lbs. cost* 500 *cents,* 1 *lb. will cost* $\frac{1}{74}$ *of* 500 *cts.* $= \frac{500}{74}$ *cts.; and* 27 *lbs. will cost* 27 *times* $\frac{500}{74}$ *cts.* $= \frac{13500}{74}$ *cts.* $= \$1.82\frac{32}{74}$.

4. If 56 pounds of tea cost $74, what is the cost of 97 pounds of the same quality? Ans. $128.17\frac{48}{56}$.

5. If 14 pair of shoes cost $30, what is the cost of 39 pair at the same rate? Ans. $83.57\frac{2}{14}$.

6. What is $\frac{5}{6}$ of 5973? Ans. $4977\frac{3}{6}$.

7. Multiply 126 by $\frac{41}{5}$. Ans. $1033\frac{1}{5}$.

8. Multiply 96374 by $\frac{153}{7}$. Ans. $2106460\frac{2}{7}$.

9. Multiply 93648 by $\frac{7}{9}$. Ans. $72837\frac{3}{9}$.

10. Multiply 496 by $\frac{83}{3}$. Ans. $8233\frac{2}{3}$.

11. If 135 bushels of wheat cost $74.25, what is the cost of 25 bushels? Ans. $13.75.

MULTIPLICATION OF FRACTIONS.

On the last page we divided by putting the divisor underneath. But it is plain that we can *suppose* it to be put underneath, *without* actually putting it there. This will bring us to the common method, which must *always* hereafter be followed.

Rule for Multiplying by Fractions.

Multiply the multiplicand by the *numerator* of the multiplier, and divide the product by the denominator.

SUMS FOR EXERCISE.

1. What is $\frac{3}{37}$ of 1975? Ans. $\frac{1}{37}$ *of* 1975 *is* $\frac{1975}{37}$; $\frac{3}{37}$ *of* 1975 *is* 3 *times* $\frac{1975}{37}$ *which is* $\frac{5925}{37}$, *&c.*

$1975 \times 3 = 5925$
$5925 \div 37 = 160\frac{5}{37}$

2. What is $\frac{19}{41}$ of 4763? Ans. $2207\frac{10}{41}$.
3. Multiply 2859 by $\frac{47}{61}$. Ans. $1658\frac{15}{61}$.
4. Multiply 962 by $\frac{171}{347}$. Ans. $474\frac{24}{347}$.
5. What is $\frac{976}{1021}$ of 37? Ans. $35\frac{377}{1021}$.
6. What is $\frac{37}{402}$ of 7631? Ans. $702\frac{143}{402}$.
7. Multiply 476 by $\frac{147}{344}$. Ans. $203\frac{140}{344}$.
8. Multiply 8691 by $\frac{102}{97}$. Ans. $9138\frac{66}{97}$.
9. Multiply 147 by $\frac{22}{8}$. Ans. $514\frac{4}{8}$.
10. Multiply 36 by $\frac{136}{7}$. Ans. $699\frac{3}{7}$.
11. Multiply 83 by $\frac{5553}{117}$. Ans. $3938\frac{70}{117}$.
12. Multiply 425 by $\frac{37}{104}$. Ans. $151\frac{31}{104}$.

13. If 8 pounds of tea cost $12, what will 3 lbs. or $\frac{3}{8}$ of that quantity cost? Ans. $4\frac{1}{2}$.

14. If a man receives $304 for 19 months' work, how much will he receive for 4 months, or $\frac{4}{19}$ of that time? Ans. $64.

15. At $2.20 a yard, what costs $\frac{3}{4}$ of a yard of cloth? Ans. $1.65.

Case II.

When the multiplier is a mixed number.

First Method.—Bring the mixed number to an improper fraction, and then proceed as usual.

SUMS FOR EXERCISE.

1. Multiply 18 by $3\frac{3}{4}$.

Ans. $3\frac{3}{4} = \frac{15}{4}$. $\frac{1}{4}$ *of* 18 *is* $\frac{18}{4}$; $\frac{15}{4}$ *of* 18 *is* 15 *times* $\frac{18}{4}$, *which is* $\frac{270}{4}$, *which equals* $67\frac{2}{4}$.

$$\begin{array}{r|r} 3\frac{3}{4} & 18 \\ 4 & 15 \\ \hline 15 & 90 \\ 4 & 18 \\ \hline & 4)270 \\ & 67\frac{2}{4} \end{array}$$

2. Multiply 127 by $3\frac{2}{3}$. Ans. $431\frac{1}{3}$.
3. Multiply 416 by $7\frac{6}{7}$. Ans. $3268\frac{4}{7}$.
4. Multiply 275 by $14\frac{5}{8}$. Ans. $4021\frac{7}{8}$.
5. Multiply 342 by $21\frac{1}{3}$. Ans. 7334.
6. Multiply 593 by $9\frac{4}{13}$. Ans. $5519\frac{8}{13}$.
7. Multiply 436 by $8\frac{11}{12}$. Ans. $3887\frac{8}{12}$.
8. Multiply 345 by $7\frac{11}{13}$. Ans. 2668.
9. Multiply 467 by $9\frac{12}{17}$. Ans. $4532\frac{11}{17}$.
10. Multiply 659 by $11\frac{14}{23}$. Ans. $7650\frac{3}{23}$.
11. Multiply 598 by $16\frac{4}{17}$. Ans. $9708\frac{12}{17}$.
12. Multiply 697 by $14\frac{4}{21}$. Ans. $9890\frac{16}{21}$.
13. Multiply 1234 by $13\frac{3}{17}$. Ans. $16259\frac{13}{17}$.
14. Multiply 37 by $41\frac{5}{7}$. Ans. $1548\frac{5}{7}$.
15. Multiply 321 by $36\frac{2}{13}$. Ans. $11598\frac{12}{13}$.
16. Multiply 64 by $121\frac{7}{8}$. Ans. 7800.
17. Multiply 235 by $16\frac{11}{15}$. Ans. $3924\frac{5}{15}$.
18. Multiply 62 by $27\frac{25}{37}$. Ans. $1715\frac{33}{37}$.
19. Multiply 261 by $14\frac{31}{120}$. Ans. $3718\frac{27}{120}$.
20. Multiply 27 by $16\frac{3}{49}$. Ans. $433\frac{6}{49}$.
21. Multiply 361 by $26\frac{39}{49}$. Ans. $9679\frac{15}{49}$.

Second Method.—Multiply, first, by the whole number; then multiply by the fraction; and then add the two products together.

SUMS FOR EXERCISE.

1. Multiply 18 by $3\frac{3}{4}$.

Ans. 3 *times* 18 *is* 54. $\frac{1}{4}$ *of* 18 *is* $\frac{18}{4}$; *and* $\frac{3}{4}$ *of* 18 *is* 3 *times* $\frac{18}{4}$, *which is* $\frac{54}{4}$, *which equals* $13\frac{3}{4}$; *and that added to the* 54 *makes* $67\frac{3}{4}$.

18	18
3	3
54	4)54
$13\frac{3}{4}$	$13\frac{3}{4}$
$67\frac{3}{4}$	

2. Multiply 25 by $5\frac{4}{7}$. Ans. $139\frac{3}{7}$.

3. Multiply 31 by $6\frac{5}{8}$. Ans. $211\frac{5}{8}$.

4. Multiply 42 by $7\frac{3}{5}$. Ans. $331\frac{3}{5}$.

5. Multiply 37 by $9\frac{4}{11}$. Ans. $346\frac{4}{11}$.

6. Multiply 231 by $8\frac{9}{16}$. Ans. $1977\frac{1}{16}$.

7. Multiply 67 by $11\frac{7}{12}$. Ans. $776\frac{1}{12}$.

8. Multiply 96 by $14\frac{6}{17}$. Ans. $1377\frac{1}{17}$.

9. Multiply 121 by $26\frac{8}{9}$. Ans. $3172\frac{8}{9}$.

10. Multiply 166 by $36\frac{4}{11}$. Ans. $6036\frac{4}{11}$.

11. Multiply 212 by $16\frac{21}{41}$. Ans. $3500\frac{24}{41}$.

12. Multiply 361 by $31\frac{7}{19}$. Ans. 11495.

13. Multiply 268 by $42\frac{10}{17}$. Ans. $11403\frac{17}{17}$.

14. Multiply 376 by $47\frac{12}{33}$. Ans. $17808\frac{24}{33}$.

15. Multiply 386 by $39\frac{72}{81}$. Ans. $15397\frac{1}{3}$.

16. Multiply 448 by $17\frac{10}{45}$. Ans. $7775\frac{13}{45}$.

PRACTICAL EXAMPLES.

1. When the price of loaf sugar is 18 cents a pound, what is the cost of $3\frac{3}{4}$ pounds? Ans. $3\frac{3}{4}$ *pounds will cost* $3\frac{3}{4}$ *times* 18 *cents.* [The pupil may finish saying it, by either the first or second method.] Ans. $67\frac{3}{4}$ cents.

2. If 1 box of oranges costs $5.25, what will $3\frac{3}{4}$ boxes cost? Ans. $19.68\frac{3}{4}$.

3. If 1 cwt. of sugar costs $9.37, what is the cost of $4\frac{1}{3}$ cwt? Ans. $43.10\frac{1}{3}$.

4. If 1 cask of raisins costs $8.25, what is the cost of $6\frac{2}{7}$ casks? Ans. $51.85\frac{5}{7}$.

5. At $6.62 a barrel, what cost $17\frac{5}{8}$ barrels of flour? Ans. $118.05\frac{4}{8}$.

6. A man bought 8 barrels of flour for $50.00. What was the cost of 3 barrels, or $\frac{3}{8}$ of them? Ans. $18.75.

7. If $10 will buy 4 barrels of cider, how many dollars will buy 7 barrels at the same rate? Ans. $17\frac{1}{2}$.

8. If I can get 5 pounds of coffee for 1 dollar, what must I pay for 75 pounds? Ans. $15.

9. At 75 cents per bushel, what cost $18\frac{4}{5}$ bushels of Indian corn? Ans. $14.10.

10. A flag staff which was 112 feet high, had $\frac{3}{8}$ of its length broken off by the wind. How long was the piece which was broken off. Ans. 42 feet.

11. A merchant owned $\frac{4}{9}$ of a vessel worth $65231, which was lost. What was the amount of his loss? Ans. $28991\frac{5}{9}$.

12. A merchant bought goods for $27268; and sold them for $\frac{7}{16}$ of the first cost. What did he gain on them, and what did he sell them for? Ans. Sold $39197\frac{12}{16}$.

13. A privateer took a prize which was valued at $38126. The captain claimed $\frac{4}{50}$ of it; the lieutenants $\frac{9}{50}$; the minor officers $\frac{7}{50}$; and the remainder was divided among the seamen. What was the share of each?

Ans. Capt. 3050\frac{4}{50}$. Lieutenants, 6100\frac{9}{50}$. Minor officers, 5337\frac{32}{50}$. Seamen 23638\frac{6}{50}$.

14. What is the cost of $48\frac{2}{7}$ yards of broad cloth, at $6.75 per yard? Ans. $325.92\frac{3}{7}$.

15. What is the cost of $37\frac{9}{16}$ pounds of tea, at 87 cents a pound? Ans. $32.67\frac{15}{16}$.

16. A merchant bought a quantity of silk for $250.00; and sold it so as to gain $\frac{3}{10}$ of the first cost. What was the gain, and what did he sell it for?

Ans. He sold it for $325.00.

17. A man bought 400 yards of cloth for $2476.00, and sold it so as to gain $\frac{4}{10}$ of the first cost. How much did he gain, and what did he sell it for?

Ans. He sold it for $3466.40.

18. A merchant bought a quantity of flour for $765, and sold it so as to gain $\frac{3}{7}$ of the first cost. How much did he sell it for? Ans. $983\frac{4}{7}$.

19. A man who has failed in business, finds that he can pay $\frac{31}{50}$ of what he owes. How much can he pay towards a debt of $475? Ans. $262\frac{43}{50}$.

20. A father wishes to divide his property to the amount of $6475; so as to give $\frac{3}{7}$ of it to his son, $\frac{3}{10}$ of it to his eldest daughter, and the rest of it to his youngest daughter. What was the share of each?

Ans. Son's, $2775. Eldest daughter's, 1942\frac{5}{10}$.
Youngest daughter's, 1757\frac{5}{10}$.

21. A merchant sold property which cost him $4362.50; so as to lose $\frac{2}{15}$ of the cost. What did he sell it for?

Ans. 3780.83\frac{5}{15}$.

22. A merchant sold a quantity of goods which cost him $765.50; so as to lose $\frac{7}{100}$ of the first cost. What did he sell it for? Ans. 711.91\frac{50}{100}$.

23. A man bought a chaise for $275, and a horse for $\frac{5}{14}$ as much. How much did he give for both?

Ans. 373\frac{3}{14}$.

RATIO.

1. If I give 10 cents for 1 pound of raisins, how many pounds can I buy for 6 cents?

Ans. *If raisins are 10 cents a pound, we know that for 1 cent we can buy $\frac{1}{10}$ of a pound; and therefore for 6 cents we can buy 6 times $\frac{1}{10}$ of a pound, which is $\frac{6}{10}$ of a pound.* $\frac{6}{10}$

☞ In the same manner the pupil may answer the following questions, taking care *always* to begin with 1.

2. If 5 cents will buy a melon, what part of a melon will 1 cent buy? What part of a melon will 4 cents buy?

3. If 3 cents buy an orange, what part of an orange will 1 cent buy? What part will 2 cents buy?

4. If 14 cents will buy a yard of calico, what part of a yard will 9 cents buy?

5. If my board costs 21 dollars a month, what part of a month will 3 dollars pay for? What part will 7 dollars pay for? What part will 18 dollars pay for?

6. If a barrel of wine can be bought for 40 dollars, what part of the barrel can be bought for 7 dollars? What for 17 dollars? What for 36 dollars?

7. If a barrel of flour costs $8.00, what part of the barrel will cost 8 cents? What part of it will cost 34 cents? What part of it will cost $3.61? What part of it will cost $5.74?

8. What part of 3 dollars is 1 dollar? 2 dollars?

9. What part of 6 pounds is 1 pound? 5 pounds?

10. What part of 9 yards is 1 yard? 7 yards?

11. What part of 4 cents is 3 cents? 6 cents?

12. What part of 30 days is 18 days? 24 days?

13. What part of 49 miles is 38 miles? 47 miles?

14. What part of 476 is 379? 435? 501?

15. If 8 yards of cloth cost $32, what will 6 yards cost?

Ans. 6 *yards is* $\frac{6}{8}$ *of* 8 *yards; and therefore will cost* $\frac{6}{8}$ *of* $32. $\frac{1}{8}$ *of* $32 *is* $\frac{32}{8}$ *of a dollar; and* $\frac{6}{8}$ *of* $32 *is* 6 *times* $\frac{32}{8}$ *of a dollar, which is* $\frac{192}{8}$ *of a dollar; which equals* $24.

$32
6
8)192
24

16. If 20 bushels of oats cost $9.60, what will be the cost of 3 bushels? Ans. $1.44.

17. If a company of 365 men consumes 75 barrels of pork in 9 months; how many barrels will 500 men consume in the same time? Ans. $102\frac{270}{365}$ barrels.

18. If 5 pounds of butter costs 75 cents, how much will 13 pounds cost? Ans. $1.95.

19. If $100 gain $6 interest in one year, how much will $57 gain in the same time? Ans. $3.42.

20. If 15 bushels of wheat cost $14.25, what will 7 bushels cost at the same rate? Ans. $6.65.

21. If 8 yards of cloth cost $20, what will 96 yards cost at the same rate? Ans. $240.

22. A laborer earned $64 in 4 months. How long will it take him to earn $304? Ans. 19 months.

23. If a town containing 23040 acres pay a land-tax of $4608, what will be the tax on a farm of 50 acres? Ans. $10.

24. A piece of muslin containing 96 yards costs $38.40. What is the cost of 8 yards of it? Ans. $3.20.

25. When 90 pounds of tea cost $47.70, what is the cost of 518 pounds? Ans. $274.54.

26. If a staff 4 feet long casts a shadow on the ground 6 feet in length, how high is a tree whose shade is 76 feet long? Ans. $50\frac{4}{6}$ feet.

27. If 3 paces, or common steps of a person, be equal to 2 yards, how many yards will 160 paces make.

Ans. $106\frac{2}{3}$ yds.

28. A garrison of 536 men have provisions for 365 days. How long will those provisions last if the garrison be increased to 1124 men? Ans. $174\frac{64}{1124}$ days.

29. If a gentleman spends $1820 in a year or 52 weeks, how much does he spend in 39 weeks? Ans. $1365.

30. If the wages of 15 weeks come to $64.19 what is a year's wages at that rate? Ans. $222.52\frac{8}{15}$.

31. If 73 men consume 235 barrels of provisions in a certain time, how many barrels would 365 men consume in the same time? Ans. 1175 barrels.

32. If a staff 6 feet long casts a shadow 9 feet; what is the height of a steeple, which, at the same time casts a shadow 198 feet? Ans. 132 feet.

33. If $5 will pay the freight of 2 cwt. 150 miles, how far may 15 cwt. be carried for the same money? Ans. 20 m.

34. A gentleman in a rail car found that he went 8 miles in 18 minutes. At what rate an hour is that. Ans. $26\frac{13}{18}$ m.

35. A and B with a stock of $820, gain $250. What is each man's share, supposing A's stock is $350, and B's $470? Ans. A's, $106.707; B's, $143.293.

36. A captain, mate, and 20 seamen, took a prize worth $3510; of which the captain takes 11 shares, the mate 5 shares, and each sailor 1 share. What must each have?

Ans. Captain $1069.50; mate $487.50; a sailor $97.50.

37. Three merchants in trade, A furnishing $240, B $360, and C $600, gain $325. What is each man's share?

Ans. A's, $65; B's, $97.50; C's, $162.50.

38. Three men do a job of work for $64. A works 10 days, B 8 days, and C 7 days. How much should each receive? Ans. A, $25.60; B, $20.48; C, $17.92.

REDUCTION OF VULGAR FRACTIONS

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. How many fourths are there in a whole one? Then how many fourths are there in $\frac{1}{2}$?

2. How many sixths are there in a whole one? Then how many sixths are there in $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$?

3. How many eighths are there in a whole one? Then how many eighths are there in $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$?

4. How many ninths are there in a whole one? Then how many ninths are in $\frac{1}{3}$? $\frac{2}{3}$?

5. How many tenths are there in a whole one? Then how many tenths are there in $\frac{1}{2}$? $\frac{1}{5}$?

6. How many twelfths are there in $\frac{1}{4}$? Ans. *In a whole one there are* $\frac{12}{12}$; *therefore in* $\frac{1}{4}$, *there must be one third as many, which is* $\frac{4}{12}$.

7. How many twelfths are equal to $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{6}$? $\frac{2}{6}$? $\frac{5}{6}$?

8. How many fifteenths are equal to $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

9. How many sixteenths are equal to $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

10. How many eighteenths are equal to $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{6}$? $\frac{5}{6}$? $\frac{5}{9}$? $\frac{1}{9}$? $\frac{2}{9}$? $\frac{4}{9}$? $\frac{7}{9}$?

11. How many twentieths are equal to $\frac{3}{4}$? $\frac{2}{5}$? $\frac{4}{5}$? $\frac{3}{10}$? $\frac{7}{10}$?

12. How many twenty-fourths are equal to $\frac{1}{3}$? $\frac{2}{3}$? $\frac{3}{8}$? $\frac{5}{8}$?

The pupil will perceive that he multiplies the numerator by the same number that he must multiply the given denominator by,

to make the required denominator. As we must multiply the denominator 3 by eight, to make *twenty-fourths*, so we multiply the numerator 2 by 8, to make the *number* of twenty-fourths. Thus, $\frac{2}{3} = \frac{16}{24}$.

13. How many $\frac{1}{10}$* are equal to $\frac{3}{5}$? $\frac{4}{5}$?
14. How many $\frac{1}{12}$ are equal to $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$?
15. How many $\frac{1}{14}$ are equal to $\frac{3}{7}$? $\frac{5}{7}$? $\frac{5}{2}$?
16. How many $\frac{1}{18}$ are equal to $\frac{2}{3}$? $\frac{5}{6}$? $\frac{2}{9}$? $\frac{5}{9}$? $\frac{7}{9}$?
17. How many $\frac{1}{24}$ are equal to $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{11}{12}$?

Now, if with the fraction $\frac{2}{3}$, we multiply both terms by 8, and make the fraction $\frac{16}{24}$; it is evident that both terms $\frac{16}{24}$ can be divided by 8, to bring the fraction back to $\frac{2}{3}$. So also, if both terms in $\frac{3}{4}$ be multiplied by 6, the fraction would be $\frac{18}{24}$, which means just as much as $\frac{3}{4}$; and, of course, if both terms in $\frac{18}{24}$ be divided by 6, the fraction would be brought back to $\frac{3}{4}$, which is equal to $\frac{18}{24}$.

18. How many thirds are there in $\frac{4}{6}$? Ans. *As we divide* $\frac{1}{6}$ *by* 3 *to make the denominator* $\frac{1}{3}$, *we must also divide the numerator* 4 *by* 2. The answer is $\frac{2}{3}$.

19. How many thirds are equal to $\frac{3}{9}$? $\frac{8}{12}$? $\frac{5}{15}$? $\frac{16}{18}$?
20. How many $\frac{1}{4}$ are equal to $\frac{2}{8}$? $\frac{9}{12}$? $\frac{8}{16}$? $\frac{15}{20}$? $\frac{6}{24}$?
21. How many $\frac{1}{5}$ are equal to $\frac{4}{10}$? $\frac{9}{15}$? $\frac{8}{10}$? $\frac{12}{15}$? $\frac{16}{20}$?
22. How many $\frac{1}{6}$ are equal to $\frac{8}{12}$? $\frac{15}{18}$? $\frac{20}{24}$? $\frac{15}{30}$? $\frac{8}{24}$?
23. Change $\frac{16}{28}$ to $\frac{1}{7}$. Ans. *As we divide the denominator by* 4 *to make* $\frac{1}{7}$, *we must divide its numerator by* 4 *also.* The answer is $\frac{4}{7}$.

24. Change $\frac{18}{48}$ to $\frac{1}{8}$. $\frac{35}{28}$ to $\frac{1}{4}$. $\frac{35}{42}$ to $\frac{1}{6}$. $\frac{15}{35}$ to $\frac{1}{7}$. $\frac{15}{20}$ to $\frac{1}{4}$. $\frac{12}{21}$ to $\frac{1}{7}$. $\frac{30}{48}$ to $\frac{1}{8}$. $\frac{27}{36}$ to $\frac{1}{4}$. $\frac{18}{36}$ to $\frac{1}{2}$. $\frac{49}{63}$ to $\frac{1}{9}$. $\frac{21}{56}$ to $\frac{1}{8}$.

* Read, *tenths*.

25. To what lower denominators can we change $\frac{2}{6}$? Ans. *We can change* $\frac{2}{6}$ *to halves, by putting* 3 *sixths together; and to thirds, by putting* 2 *sixths together.*

26. To what lower denominators can we change $\frac{2}{4}$? $\frac{2}{8}$? $\frac{2}{9}$? $\frac{2}{10}$? $\frac{2}{12}$? $\frac{2}{14}$? $\frac{2}{15}$? $\frac{2}{16}$? $\frac{2}{18}$? $\frac{2}{20}$? $\frac{2}{21}$? $\frac{2}{24}$? $\frac{2}{27}$? $\frac{2}{30}$? $\frac{2}{32}$? $\frac{2}{48}$? $\frac{2}{63}$? $\frac{2}{72}$? $\frac{2}{81}$? $\frac{2}{108}$?

It is always best to reduce a fraction to as small a denominator as it can be changed to. Thus, $\frac{1}{2}$ is better than $\frac{2}{4}$; $\frac{6}{8}$ is better than $\frac{8}{12}$; $\frac{3}{4}$ is better than $\frac{15}{20}$; &c. In this last example, although $\frac{15}{20}$ can be changed to *halves*, yet $\frac{15}{20}$ will not make a whole number of halves; but they will make a whole number of *fourths*. So, although $\frac{3}{4}$ may be changed to $\frac{6}{8}$, $\frac{9}{12}$, $\frac{12}{16}$, $\frac{15}{20}$, and $\frac{18}{24}$; yet $\frac{15}{24}$ will not make just a half, nor a whole number of $\frac{3}{6}$, $\frac{4}{8}$, or $\frac{5}{10}$. But it will make just $\frac{5}{8}$. Therefore, when $\frac{15}{24}$ are reduced to $\frac{5}{8}$, we say it is reduced to its *lowest terms*.

27. Reduce to their lowest terms $\frac{2}{4}$, $\frac{6}{9}$, $\frac{9}{15}$, $\frac{6}{18}$, $\frac{14}{35}$, $\frac{7}{42}$, $\frac{26}{65}$, $\frac{10}{14}$, $\frac{56}{72}$, $\frac{27}{72}$, $\frac{18}{90}$, $\frac{21}{56}$, $\frac{64}{96}$, $\frac{42}{80}$, $\frac{36}{48}$, $\frac{16}{24}$, $\frac{35}{65}$, $\frac{42}{48}$, $\frac{72}{80}$.

INDUCTIVE EXERCISES FOR THE SLATE.

1. The learner may first commence at the remarks after No. 17 in the preceding exercises; and perform the sums on his slate, excepting questions 25 and 26.

2. Reduce $\frac{25}{75}$ to its lowest terms.

Operation. { Divisors. $\div 5 \div 5$ } $\frac{25}{75} = \frac{5}{15} = \frac{1}{3}$. = Ans.

3. Reduce $\frac{512}{1000}$ to its lowest terms.

Divisors $\div 4 \div 8 \div 8 \div 2$

$\frac{512}{1000} = \frac{128}{1024} = \frac{16}{128} = \frac{2}{16} = \frac{1}{8}$, the answer.

4. Reduce $\frac{324}{378}$ to its lowest terms. Ans. $\frac{6}{7}$.

5. Reduce $\frac{81}{720}$ to its lowest terms. Ans. $\frac{1}{9}$.
6. Reduce $\frac{216}{288}$ to its lowest terms. Ans. $\frac{3}{4}$.
7. Reduce $\frac{182}{392}$ to its lowest terms. Ans. $\frac{13}{28}$.
8. Reduce $\frac{468}{1164}$ to its lowest terms. Ans. $\frac{117}{291}$.

DEFINITION AND RULE.

REDUCTION of Fractions is the changing of the *form* of a fractional quantity, without altering its value.

RULE.—Divide both the numerator and the denominator by any number that will divide each without a remainder; and this will form a new fraction of the same value as the first.

Divide the terms of this last fraction in the same way, if it can be done; and so on, till no number will divide them both.

Remarks.—As it is sometimes difficult to find what number will divide the terms of fractions, the following directions may be of some use:

1. If the right hand figure of any number is 0, the number is divisible by 10, and of course by 5 or 2.

2. If the two right hand figures are 00, the number is divisible by 100, and of course by 10, 5, or 4, (because $4 \times 25 = 100$) or 2.

3. If the right hand figure can be divided by two, the number is divisible by 2.

4. If the right hand figure can be divided by 5, the number is divisible by 5.

5. As hundreds are divisible by 4; if the remainder of hundreds (i. e. the two *right hand figures*,) are divisible by 4, the number may be divided by 4.

6. Even 100s are divisible by 8, (because $8 \times 25 = 200$.) Therefore, if the remainder of even 100s (i. e. the two right hand figures,) are divisible by 8, the number may be divided by 8, and of course by 4 or 2.

7. If all the figures of a number, added together, are divisible by 3 or 9, the number is divisible by 3 or 9.

8. If the sum of the first, third, fifth, &c. figures of any number, are equal to the sum of the second, fourth, sixth, &c., that number is divisible by 11.

9. When a number is divisible by either 2 or 3, it is divisible by their product 6. And so if it is divisible by either 4 or 3, it is also by 12.

Examples.

Tell what the following numbers can be divided by; and how you know.

470, 5500, 474, 895, 1072, 4640, 468, 735, 432, 4765, 8476, 6764, 18848, 76232, 7560, 75465, 897640, 6471660, 654577, 75432137, 674311.

APPLICATION OF THE RULE.

1. Reduce $\frac{100}{1600}$ to its lowest terms. Ans. $\frac{1}{16}$.
2. Reduce $\frac{1296}{3024}$ to its lowest terms. Ans. $\frac{3}{7}$.
3. Reduce $\frac{1426}{2458}$ to its lowest terms. Ans. $\frac{713}{1429}$.
4. Reduce $\frac{164}{328}$ to its lowest terms. Ans. $\frac{82}{164}$.
5. Reduce $\frac{2640}{2880}$ to its lowest terms. Ans. $\frac{11}{12}$.
6. Reduce $\frac{160}{168}$ to its lowest terms. Ans. $\frac{20}{21}$.
7. Reduce $\frac{1470}{2205}$ to its lowest terms. Ans. $\frac{2}{3}$.
8. Reduce $\frac{105}{455}$ to its lowest terms. Ans. $\frac{3}{7}$.
9. Reduce $\frac{486}{9720}$ to its lowest terms. Ans. $\frac{1}{20}$.
10. Reduce $\frac{468}{1184}$ to its lowest terms. Ans. $\frac{117}{296}$.
11. Reduce $\frac{806}{42315}$ to its lowest terms. Ans. $\frac{2}{105}$

12. Reduce $\frac{120}{184}$ to its lowest terms. Ans. $\frac{15}{23}$
13. Reduce $\frac{825}{880}$ to its lowest terms. Ans. $\frac{15}{16}$
14. Reduce $\frac{192}{576}$ to its lowest terms. Ans. $\frac{1}{3}$
15. Reduce $\frac{72}{96}$ to its lowest terms. Ans. $\frac{3}{4}$
16. Reduce $\frac{875}{1125}$ to its lowest terms. Ans. $\frac{7}{9}$
17. Reduce $\frac{5184}{6912}$ to its lowest terms. Ans. $\frac{3}{4}$
18. Reduce $\frac{25020}{46320}$ to its lowest terms. Ans. $\frac{9}{14}$
19. Reduce $\frac{20160}{21840}$ to its lowest terms. Ans. $\frac{7}{13}$.
20. Reduce $\frac{25020}{138112}$ to its lowest terms. Ans. $\frac{10}{61}$.
21. Reduce $\frac{544432}{681128}$ to its lowest terms. Ans. $\frac{21}{34}$.
22. Reduce $\frac{26136}{60984}$ to its lowest terms. Ans. $\frac{3}{7}$.

MISCELLANEOUS EXERCISES IN FRACTIONS.

1. Reduce $9\frac{6}{8}$ to an improper fraction. Ans. $\frac{78}{8}$.
2. Reduce $\frac{1101}{27}$ to a mixed number. Ans. $44\frac{3}{27}$.
3. Add together $7\frac{4}{9}$, $12\frac{5}{9}$, $14\frac{8}{9}$, $21\frac{7}{9}$, $6\frac{1}{9}$, $\frac{2}{9}$. Ans. 63.
4. Reduce $71\frac{9}{35}$ to an improper fraction. Ans. $\frac{2494}{35}$.
5. Subtract $4\frac{11}{18}$ from $9\frac{7}{18}$ Ans. $4\frac{7}{9}$.
6. Multiply $9\frac{3}{4}$ by 7. Ans. $68\frac{1}{4}$.
7. Reduce $\frac{2852}{33}$ to a mixed number. Ans. $86\frac{14}{33}$.
8. Add together $14\frac{2}{27}$, $8\frac{3}{27}$, $5\frac{6}{27}$, $10\frac{2}{27}$, $\frac{15}{27}$. Ans. $38\frac{7}{27}$.
9. Subtract $11\frac{11}{24}$ from $13\frac{5}{24}$. Ans. $1\frac{3}{4}$.
10. Divide $88\frac{4}{7}$ by 5. Ans. $17\frac{5}{7}$.
11. Multiply $14\frac{2}{7}$ by 8. Ans. $114\frac{2}{7}$.
12. Multiply $27\frac{18}{19}$ by 9. Ans. $251\frac{10}{19}$.
13. Subtract $12\frac{9}{25}$ from $18\frac{3}{25}$. Ans. $5\frac{19}{25}$.
14. Reduce $109\frac{7}{11}$ to an improper fraction. Ans. $\frac{1206}{11}$.
15. Divide $97\frac{3}{5}$ by 6. Ans. $16\frac{3}{5}$.
16. Divide $104\frac{3}{19}$ by 7. Ans. $14\frac{9}{19}$

17. Divide $137\frac{1}{13}$ by 9. Ans. $15\frac{3}{13}$.

18. Subtract $16\frac{21}{20}$ from $27\frac{3}{20}$. Ans. $10\frac{11}{20}$.

19. Subtract $36\frac{41}{80}$ from $94\frac{7}{80}$. Ans. $57\frac{33}{80}$.

20. Reduce $\frac{396}{1152}$ to its lowest terms. Ans. $\frac{7}{9}$.

21. Reduce $23\frac{17}{35}$ to an improper fraction. Ans. $\frac{822}{35}$.

22. Reduce $\frac{3446}{47}$ to a mixed number. Ans. $73\frac{15}{47}$.

23. Multiply 67 by $9\frac{1}{12}$. Ans. $608\frac{7}{12}$.

24. Reduce $183\frac{5}{21}$ to an improper fraction. Ans. $\frac{3848}{21}$.

25. Add $\frac{4}{25}$, $\frac{8}{25}$, $\frac{13}{25}$, $\frac{16}{25}$, and $\frac{19}{25}$ together. Ans. $2\frac{4}{5}$.

26. Add $13\frac{1}{15}$, $9\frac{4}{15}$, and $3\frac{7}{15}$ together. Ans. $25\frac{4}{5}$.

27. Multiply 126 by $8\frac{1}{2}$. Ans. $1033\frac{1}{2}$.

28. Multiply 96374 by $21\frac{6}{7}$. Ans. $2106460\frac{2}{7}$.

29. Multiply $410\frac{5}{12}$ by 6. Ans. $2462\frac{1}{2}$.

Reducing the Ratio, or Canceling.

30. If 8 pencils cost 63 cents, what will 24 pencils cost?

Ans. 24 *pencils will cost* $\frac{24}{8}$ *of* 63 *cents; which equals* 3 *times* 63 *cents, which equals* 189 *cents, or* $1.89.

$\frac{24}{8}$ of 63 = 3 times 63

63

189

31. If 20 yards of cloth cost $40, what will 8 yards cost?

Ans. 8 *yards will cost* $\frac{8}{20}$ *of* $40; *which equals* $\frac{2}{5}$ *of* 40; *which equals* $16.

$\frac{8}{20}$ of 40 = $\frac{2}{5}$ of 40

2

5)80

16

32. If 8 pair of shoes cost $20 dollars what will 96 pair come to? Ans. $240.

33. If 24 penknives cost $27, what will be the cost of 60 penknives. Ans. $67.50.

34. If 5 bushels of wheat cost $7.50, what will be the cost of 240 bushels? Ans. $360.

35. If 3 ounces of silver cost 17 shillings; what will 48 ounces cost? Ans. 272 shillings.

36. If $3.20 buy 8 yards of cloth, how much can be bought for $38.40? Ans. 96 yards.

37. If 72 yards of cambric cost $119.44, how much will 9 yards come to? Ans. $14.93.

38. If 20 gallons of water weigh 167 pounds, what will 180 gallons weigh? Ans. 1503 pounds.

39. If a staff which is 3 feet long, casts a shadow 9 feet, how high is a steeple whose shadow at the same time is 285 feet? Ans. 95 feet.

40. A and B have a joint stock of $2100, of which A put in $1800, and B $300. They gain in a year $1000. What is each one's share of the profits? ☞ $\frac{1800}{2100} = \frac{6}{7}$; and $\frac{300}{2100} = \frac{1}{7}$. Ans. A's, $857.14; B's, $142.85.

41. Three partners, C, D, and E, trading together gained $120. C's stock was $280, D's $600, E's $320. What was the share of each? Ans. C's, $28; D's, $60; E's, $32.

42. Three merchants freighted a vessel with 2160 barrels of flour; of which F owned 960, G 720, and H 480. In a storm the seamen were obliged to throw overboard 900 barrels. How many barrels did each lose?

Ans. F lost 400 barrels; G, 300; H, 200.

43. Three butchers hire a pasture field for $96, and put into it 300 beef cattle. As 80 of these belonged to A, 100 to B, and 120 to C, how much had each to pay?

Ans. $25.60, $32, $38.40.

44. Three gentlemen purchased a tract of land, for which A paid $2880, B paid $11520, and C paid $4800. They afterwards sold the land for $1920. What is each man's share of the avails? Ans. A, $288; B, $1152; C, $480.

DENOMINATE FRACTIONS.

All the various measuring quantities and their divisions, are called *denominations*.

The following are the principal tables of the different denominations.

1. Long Measure.

12 inches, (in.) = 1 foot, (ft.)	1 rod = $\frac{1}{320}$ of 1 m.
3 feet, = 1 yard, (yd.)	1 yd. = $\frac{2}{11}$ of 1 r.
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards = 1 rod, (r.) or pole, (p.) or perch.	1 ft. = $\frac{2}{33}$ of 1 r, or $\frac{1}{3}$ of 1 yd.
320 rods, = 1 mile, (m.)	1 in. = $\frac{1}{12}$ of 1 ft.

A furlong is $\frac{1}{8}$ of a mile, which = 40 rods.

A league is equal to 3 miles.

A hand, (for measuring height of horses,) = 4 inches.

A fathom, (for measuring depths,) = 6 feet.

Surveyors measure distance on land with a chain that is 4 rods or 66 feet long. It is divided into 100 links.

2. Cloth Measure.

4 nails, (nl.) = 1 quarter (qr.)	1 nail = $\frac{1}{4}$ of a qr.
4 quarters = 1 yard.	$2\frac{1}{4}$ in. = 1 nail.

An Ell Flemish, (E. F.) = 3 qrs. of a yard.
Ell English, (E. E.) = 5 qrs.
Ell French, or aune = 6 qrs.

3. Square Measure.

144 sq. in. = 1 sq. foot.
9 sq. feet = 1 sq. yard.
30¼ sq. yds. or 272¼ sq. feet. } = 1 sq. rod.
40 sq. rods = 1 rood.
4 roods, or 160 sq. rods } = 1 acre, (a.)

640 acres = 1 sq. mile.
1 rood = ¼ of an acre.
1 rod = $\begin{cases} \frac{1}{40} \text{ of 1 rood or} \\ \frac{1}{160} \text{ of 1 acre.} \end{cases}$
1 sq. yd. = $\frac{4}{121}$ of 1 rd.
1 sq. ft. = $\frac{1}{9}$ of 1 sq. yd.
1 sq. in. = $\frac{1}{144}$ of 1 sq. ft.

A square chain = 16 sq. rods; or $\frac{1}{10}$ of an acre.

4. Solid or Cubic Measure.

1728 solid in. = 1 solid foot.
27 solid ft. = 1 solid yard.
16 solid ft. = 1 ft. of wood.
8 ft. of wood = 1 cord.

1 sol. ft. = $\frac{1}{27}$ of 1 yd.
1 sol. in. = $\frac{1}{1728}$ of 1 ft.
128 solid feet = 1 cord.
1 sol. ft = $\frac{1}{128}$ of a cord.

40 feet of round timber, or 50 ft. of hewn = 1 ton.

A cord is generally 8 ft. long, 4 ft. wide, and 4 ft. high.

5. Liquid Measure.

4 gills = 1 pint, (pt.)
2 pints = 1 quart, (qt.)
4 quarts = 1 gallon, (gal.)

1 quart = ¼ of 1 gal.
1 pint = ½ of 1 qt.
1 gill = ¼ of 1 pt.

We frequently hear that a *hogshead* = 63 gallons; a *barrel*, = 31½ gallons; a *pipe* or *butt*, = 2 hogsheads; &c. But these are rather *names* of the casks in which liquors are imported. It is the practice to gauge all such vessels, and to charge according to their actual contents.

6. Dry Measure.

2 pints = 1 quart.
8 quarts = 1 peck, (pk.)
4 pecks = 1 bushel, (bush.)

1 pk. = ¼ of 1 bush.
1 qt. = ⅛ of 1 pk.
1 pt. = ½ of 1 qt.

7. Avoirdupois Weight.

16 drams = 1 ounce, (oz.)	1 C. = $\frac{1}{20}$ of 1 ton.
16 ounces = 1 pound, (lb.)	1 lb. = $\frac{1}{25}$ of 1 qr. or $\frac{1}{100}$ of C.
25 pounds = 1 qr. of a hundred.	
4 quarters = 1 hundred, (C.)	1 oz. = $\frac{1}{16}$ of 1 lb.
20 hundred = 1 ton (T.)	1 dram = $\frac{1}{16}$ 1 oz.

A hundred weight, (cwt.) which = 112 lbs. is used in some places. Then, 1 qr. of cwt. = 28 lbs.

A hundred weight of fish is called a *quintal.*

8. Troy Weight.

24 grains, (gr.) = 1 pennyweight, (dwt.)	1. oz = $\frac{1}{12}$ of 1 lb.
20 pennyweights = 1 ounce.	1 dwt. = $\frac{1}{20}$ of 1 oz.
12 ounces = 1 pound.	1 gr. = $\frac{1}{24}$ of 1 dwt.

The pound Troy is used chiefly as a standard. It is rarely mentioned in business. Thus, we frequently hear of 30, 40, or 50 oz. of silver.

9. Apothecaries' Weight.

20 Troy grains = 1 scruple, (℈.)	1 dram = $\frac{1}{8}$ of 1 ℥.
3 scruples = 1 dram, (ʒ.)	1 scruple = $\frac{1}{3}$ of 1 ʒ.
8 drams = 1 Troy ounce, (℥.)	1 grain = $\frac{1}{20}$ of 1 ℈.

This weight is used merely for mixing medicines according to prescriptions. Druggists buy and sell by avoirdupois weight.

10. Time.

60 seconds, (sec.) = 1 minute, (min.)	1 yr. = $\frac{1}{100}$ of 1 cen.
60 minutes = 1 hour, (h.)	1 day = $\frac{1}{365}$ of 1 yr.
24 hours = 1 day, (d.)	1 hour = $\frac{1}{24}$ of 1 d.
365 days = 1 year, (y.)	1 min. = $\frac{1}{60}$ of 1 h.
365 days and 6 h. = 1 Julian year.*	1 sec. = $\frac{1}{60}$ of 1 min
100 years = 1 century, (cen.)	

* Used when the time is as much as four years.

4 weeks are nearly a month; and 52 weeks are nearly a year. Also, 7 days = 1 week.

The year is divided into 12 months, called *calendar months*, because they were written in a calendar or almanac. For the number of days in each, learn the following:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
Which hath four and twenty-four,
And every leap-year, one day more.

We reckon 365 days as a year; and then in 4 years, the odd six hours make 24 hours, which is one day. On this account, every fourth year, we reckon in February, 29 days instead of 28; and make the year 366 days long.

Leap-year always happens when we can divide the year by 4, without a remainder.

11. Circular Measure.

60 seconds (″) = 1 minute, (′)	1° = $\frac{1}{360}$ of a circle.
60 minutes = 1 degree, (°)	1′ = $\frac{1}{60}$ of 1°.
360 degrees = 1 circle, (C.)	1″ = $\frac{1}{60}$ of 1′.

Also, 30 degrees = 1 sign; 12 signs = 1 circle of the zodiac.

A degree of the earth's surface is about 69$\frac{1}{5}$ miles.

Sailors and geographers sometimes call a minute a mile, and a degree 60 miles, instead of 69$\frac{1}{5}$.

12. Money, or Currency.

People of almost every country have their peculiar methods of computing their money. In the United States, the original currency was the English or sterling money. But since the revolution, a new currency has been established, called the *federal currency*, or *federal money*. But owing to long practice in sterling money, we still, in some measure, retain its denominations in common conversation. This, however, ought to be dispensed with.

Federal Money.

10 mills, (m.) = 1 cent, (ct.)
10 cents = 1 dime, (d.)
10 dimes or 100 cts. = 1 dollar, ($)
10 dollars = 1 eagle.

1 dol. = $\frac{1}{10}$ of 1 eagle.
1 dime = $\frac{1}{10}$ of 1 dol.
1 ct. = $\frac{1}{10}$ of 1 dime or $\frac{1}{100}$ of 1 dol.
1 mill = $\frac{1}{10}$ of 1 ct.

Sterling Money.

4 farthings (qr.) = 1 penny, (d.)
12 pence = 1 shilling, (s.)
20 shillings = 1 pound, (£.)

1 shilling = $\frac{1}{20}$ of 1 £.
1 penny = $\frac{1}{12}$ of 1 s.
1 farthing = $\frac{1}{4}$ of 1 d.

Farthings are generally written as fourths of a penny; as, 6¼d. instead of 6d. 3 qrs.

State Currencies.

In Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, Virginia, Kentucky, Tennessee,	16⅔ cents is called a shilling; and 6 shillings make a dollar.
New York, Ohio, North Carolina,	12½ cents is called a shilling; and 8 shillings make a dollar.
New Jersey, Pennsylvania, Delaware, Maryland,	13⅓ cents is called a shilling; and 7s. 6d. make a dollar.
South Carolina, Georgia,	21 cts. 4 m. is called a shilling; and 4s. 8d. make a dollar.
In Canada, Nova Scotia, New Brunswick, &c.,	1 shilling is 20 cents; 5 shillings 1 dollar.

REDUCTION OF DENOMINATE FRACTIONS.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. How many pints are in 1 quart? 2 quarts? 3 quarts?
2. How many quarts are in 6 pints? 7 pints?
3. How many quarts are in 4 pints? 9 pints?
4. How many pints in 4 quarts 1 pint? 3 quarts 1 pint?
5. What is $\frac{2}{4}$ of a pint? $\frac{3}{4}$ of a pint?
6. What part of a pint is 1 gill? 2 gills? 3 gills?
7. How many gills are in 1 pint? 3 pints? 5 pints?
8. How many gills are in 4 pints 3 gills? 5 pints 2 gills? 6 pints 1 gill?
9. How many pints are there in 4 gills? 7 gills? 10 gills? 13 gills? 17 gills? 23 gills? 36 gills?
10. What is $\frac{1}{4}$ of a gallon called?
11. What is $\frac{2}{4}$ of a gallon? $\frac{3}{4}$ of a gallon?
12. What part of a gallon is 1 quart? 2 quarts? 3 quarts?
13. How many quarts are there in 1 gallon 3 quarts? 5 gallons 1 quart? 7 gallons 3 quarts? 9 gallons 3 quarts?
14. How many gallons are there in 5 quarts? 7 quarts? 14 quarts? 21 quarts? 28 quarts? 35 quarts? 42 quarts?
15. What is $\frac{1}{16}$ part of a pound called?
16. What is $\frac{2}{16}$ of a pound? $\frac{3}{16}$ of a pound?
17. What is $\frac{4}{16}$ of a pound? $\frac{5}{16}$ of a pound?
18. What is $\frac{6}{16}$ of a pound? $\frac{1}{16}$ of a pound?
19. What is $\frac{8}{16}$ of a pound? $\frac{1}{16}$ of a pound?
20. What is $\frac{13}{16}$ of a pound? $\frac{11}{16}$ of a pound?

21. What part of a pound is 3 ounces? 5 ounces? 10 ounces? 1 ounce? 9 ounces? 4 ounces? 12 ounces? 2 ounces? 14 ounces? 6 ounces? 11 ounces? 7 ounces? 13 ounces? 8 ounces?

22. How many ounces are there in 1 pound? 2 pounds? 3 pounds? 1 pound 4 ounces? 2 pounds 10 ounces? 3 pounds 14 ounces?

23. How many pounds are there in 16 ounces? 20 ounces? 32 ounces? 38 ounces? 29 ounces? 41 ounces? 31 ounces?

24. What is $\frac{1}{12}$ of a foot? Ans. *One inch.*

25. What is $\frac{2}{12}$ of a foot? What is $\frac{3}{12}$ of a foot?

26. What is $\frac{10}{12}$ of a foot? What is $\frac{11}{12}$ of a foot?

27. What part of a foot is 5 inches? 7 inches? 3 inches? 8 inches? 11 inches? 9 inches? 4 inches? 10 inches? 6 inches?

28. How many inches are there in 1 foot? 2 feet? 1 foot and 7 inches? 2 feet and 10 inches? 3 feet and 5 inches?

29. How many feet are there in 12 inches? In 24 inches? In 17 inches? In 35 inches? In 46 inches? In 67 inches?

30. What is $\frac{1}{24}$ of a day called? $\frac{2}{24}$ of a day? $\frac{7}{24}$ of a day? $\frac{13}{24}$ of a day? $\frac{18}{24}$ of a day? $\frac{21}{24}$ of a day?

31. What part of a day is 1 hour? 3 hours? 7 hours? 10 hours? 15 hours? 21 hours?

32. How many hours are there in 1 day 6 hours? 2 days 8 hours? 4 days 9 hours?

33. How many days are there in 36 hours? 48 hours? 54 hours? 60 hours?

34. What part of a yard is 2 feet? 3 feet?

35. How many yards are there in 7 feet? 12 feet? 16 feet? 31 feet? 27 feet?

36. How many feet are there in 2 yards 2 feet? 3 yards 1 foot? 4 yards 2 feet? 7 yards 1 foot?

37. How many yards in 8 quarters? 14 quarters? 18 quarters? 23 quarters? 30 quarters?

38. How many quarters in 2 yards 3 quarters? 4 yards 1 quarter? 7 yards 2 quarters? 10 yards 1 quarter? 12 yards 3 quarters?

39. What is $\frac{2}{4}$ of a quarter of a yard? $\frac{3}{4}$?

40. What part of a quarter is 1 nail? 2 nails? 3 nails?

41. How many quarters are 7 nails? 18 nails? 22 nails? 31 nails? 43 nails?

42. How many nails are there in 3 quarters 1 nail? 5 quarters 2 nails? 7 quarters 3 nails? 9 quarters 2 nails?

43. What is $\frac{1}{4}$ of a bushel called?

44. What is $\frac{2}{4}$ of a bushel? $\frac{3}{4}$ of a bushel?

45. What part of a bushel is 1 peck? 2 pecks? 3 pecks?

46. How many pecks are there in 1 bushel? 1 bushel 3 pecks? 2 bushels 1 peck? 3 bushels 2 pecks? 4 bushels 1 peck?

47. How many bushels are there in 4 pecks? 9 pecks? 26 pecks? 17 pecks? 35 pecks? 43 pecks?

48. How many quarts are there in 1 peck? 2 pecks 7 quarts? 3 pecks 4 quarts? 7 pecks 3 quarts? 8 pecks 6 quarts?

49. How many pecks are there in 8 quarts? 17 quarts? 27 quarts? 39 quarts? 47 quarts? 52 quarts?

INDUCTIVE EXERCISES FOR THE SLATE.

1. In 642 inches, how many feet? Ans. *There are as many feet as there are collections of* 12 *inches in* 642 *inches. Therefore, there are* 53 *feet, and* 6 *inches over.*

2. In the 53 feet of the last answer, how many yards? Ans. 17 yards and 2 feet.

3. Then, putting the first and second questions together; in 642 inches, how many yards?

☞ To be said like *two* sums.

The pupil must *always* be made to write the *name* of the denominations after every quantity.

12)642 inches.
3)53 ft. 6 in.
17 yds. 2 ft. 6 in.

4. In 47 pints, how many quarts? Ans. 23 qts. and 1 pt.

5. In 23 quarts and 1 pint, how many gallons?
Ans. 5 gal. and 3 qts. 1 pt.

6. Then in 47 pints, how many gallons?
Ans. 5 gals. 3 qts. and 1 pt.

7. In 63 gills, how many pints?
Ans. 15 pts. and 3 gills over.

8. In 15 pints and 3 gills, how many quarts?
Ans. 7 qts. and 1 pt. 3 gills over.

9. In 7 quarts and 1 pint, how many gallons?
Ans. 1 gal. and 3 qts. 1 pt. over

10. Then in 63 gills, how many gallons?
Ans. 1 gal. and 3 qts. 1pt. 3 gills over.

11. In 536 pints of corn, how many quarts?
Ans. 268 qts.

12. In 268 quarts of corn, how many pecks?
Ans. 33 pks. 4 qts.

13. In 33 pecks 4 quarts, how many bushels?
Ans. 8 bush. 1 pk. and 4 qts.

14. Then in 536 pints of corn, how many bushels?
Ans. 8 bush. and 1 pk. 4 qts. over.

15. In 4032 ounces, how many pounds? And in those pounds, how many quarters of a hundred weight?
Ans. 9 qrs. of cwt.

16. In 3763 inches, how many feet?
Ans. 313 ft. and 7 in.

17. In 313 feet 7 inches, how many yards?
Ans. 104 yds. 1 ft. 7 in.

18. In 56789 pence, how many shillings?
Ans. 4732s. 5d.

19. In 4732s. 5d. how many pounds?
Ans. 236£. 12s. 5d.

20. In 490 pence, how many pounds? Ans. 2£. 0s. 10d.

21. In 7 gallons, how many quarts? Ans. *In* 1 *gallon there are* 4 *quarts; therefore, in* 7 *gallons there are* 7 *times* 4 *quarts, which is* 28 *quarts.*

22. In 28 quarts, how many pints? Ans. 56 pts.

23. In 56 pints, how many gills? Ans. 224 gills.

24. In 81 bushels how many pecks? Ans. 324 pks.

25. In 324 pks. how many quarts? Ans. 2592 qts.

26. In 2592 qts. how many pints? Ans. 5184 pts.

27. In 79 bushels, how many pecks? Ans. 316 pks.

28. Then, in 79 bushels, how many quarts?
☞ First, how many pecks. Ans. 2528 qts.

29. In 72 yds. how many quarters? Ans. 288 qrs

30. Then in 72 yards, how many nails? Ans. 1152 nls.

31. In 78£. how many shillings? And in those shillings, how many pence? Ans. 18720d.

32. In 8 rods, how many feet? And in those feet, how many inches? Ans. 1584 in.

33. In 64 cwt. how many qrs. of a cwt.? And in those qrs. how many lbs.? And in these lbs. how many oz.? Ans. 114688 oz.

34. In 86 days, how many hours? And in those hours, how many minutes? And in those minutes, how many seconds? Ans. 7430400 sec.

35. In 637 bushels, how many pks.? How many qts.? Ans. 20384 qts.

36. How many qrs. in 2 yds. 1 qr.? How many nails? Ans. 36 nls.

37. In 231 gallons how many gills?

Ans. As gallons are first divided into quarts, I must first see how many qts. are in 231 gallons. There are 231 times 4 qts. which are 924 qts. As quarts are divided into pints, I will see how many pts. are in the 924 qts. There are 924 times 2 pts. which equals 1848 pts.; and in 1848 pts. there are 1848 times 4 gls. = 7392 gls.

231 gals.
4
924 qts.
2
1848 pts.
4
7392 gls.

DEFINITIONS.

1. REDUCTION is the changing of a quantity from one denomination to another, without altering its value.

2. In reducing a number from one denomination to another, we must first consider whether the denomination required, expresses a greater or a smaller quantity than the denomination given.

3. If the required denomination expresses a greater quantity than the denomination given, the operation is called *Reduction Ascending*, and the sum is to be performed by *Division*.

The reason for this is evident. For it is plain that if we have the length of a line in *inches*, it cannot contain as many *feet*; if we have the weight of a thing in *ounces*, it must weigh a less number of *pounds*.

4. If the required denomination expresses a smaller quantity than the denomination given, the operation is called *Reduction Descending*; and the sum is to be performed by *Multiplication*.

If a thing weighs a certain number of *pounds*, it must weigh a greater number of *ounces*; if it measures a certain number of *bushels*, it must measure a greater number of *pecks*.

RULE.

To change a number from a lower denomination to a higher,

First, See what is the *next* higher denomination, which, if not the required denomination, is a part of it.

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1. What is Reduction? 2. What is the first thing to be considered when we wish to change a quantity from one denomination to another? 3. Supposing the required denomination is greater than the denomination given? Why is this? 4. Supposing the required denomination smaller than the one given? Explain. What is the first step in changing a number from a lower denomination to a higher?

Second, Divide by as many as it takes of the given denomination to equal *one* in the next higher. The quotient will express the value of the quantity in that next denomination; and the remainder, if any, will retain the name of the denomination divided.

Third, If the quotient just found, is not in the required denomination, consider this new denomination as the given one; and change it, in the same manner as before, (without disturbing the remainder,) into the next higher.

Fourth, Proceed in the same manner, till you have obtained the required denomination. And the last quotient, together with the several remainders, will be the proper answer.

NOTE.—The *required* denomination, if not mentioned, is that which is usually called the *unit*.

RULE.

To change a number from a higher denomination to a lower,

First, See what is the *next* lower denomination.

Second, Multiply by as many as it takes of that next denomination to make one in the given denomination; and to the product, add as many of that lower denomination as are expressed in the given quantity.

Third, If the product just found is not in the required denomination, consider this *new* denomination as the *given* one; and change *it*, in the same manner as before, to the next lower.

Fourth, Proceed in the same manner, till you come to the required denomination.

NOTE.—The *required* denomination, if not mentioned, is the *lowest that is mentioned* in the given quantity.

What is the second? Third? Fourth? What is the required denomination? What is the first step in changing a number from a higher to a lower denomination? What is the second? Third? Fourth? What is the required denomination in reduction descending?

APPLICATION OF THE RULE.

1. In 5437 lbs. how many cwt.?

☞ See page 120. Ans. 48 cwt. 2 qrs. 5 lb.

2. In 535 gills, how many gallons?
Ans. 16 gals. 2 qts. 1 pt. 3 gls.

3. In 7531 grains, how many pounds Troy weight?
Ans. 1 lb. 3 oz. 13 dwt. 19 grs.

4. In 396 rods, how many miles? Ans. 1 m. 76 rds.

5. In 49 sq. ft. how many sq. in.? Ans. 7056 sq. in.

6. In 394 gills, how many gals.?
Ans. 12 gals. 1 qt. 2 gls.

7. In 57 cords 116 feet, how many solid feet?
Ans. 7412 sol. ft.

8. In 54000 sec. how many degrees? Ans. 15 deg.

9. In 4768 in. how many yards?
Ans. 132 yds. 1 ft. 4 in.

10. In 132 yards, how many rods? Ans. There are as many rods as there are collections of $5\frac{1}{2}$ yds. in 132 yds. In performing this, it is difficult to divide by $5\frac{1}{2}$. On this account, we say 11 *half yards* = 1 rod; and as 132 yds. = $\frac{264}{2}$ of a yard, we may see how often 11 half yds. are contained in 264 half yds. The answer is 24 rods.

11. How many seconds are there in 5 years, allowing $365\frac{1}{4}$ days to the year? Ans. 157788000 sec.

12. In 26744 seconds, how many hours?
Ans. 7 h. 25 min. 44 sec.

13. In 974 yrs. how many seconds?
Ans. 30737102400 sec.

14. In 6725 nails, how many yards?
Ans. 420 yds. 1 qr. 1 n.

15. In 1234 yds. how many qrs.? Ans. 4936 qrs.

16. In 6 lb. 1 oz. how many oz.? Ans. 97 oz.

17. In 147 gals. 2 qts. how many qts.? Ans. 590 qts.

18. In 767 lbs. 4 oz. how many oz. Troy?
Ans. 9208 oz.

19. How many pounds in 5520 pence? Ans. 23£
20. In 17 lbs. how many scruples? Ans. 4896 scr
21. In 9 cwt. 2 qrs. 8 lbs. how many lbs.?
Ans. 1072 lbs
22. In 16 yds. 1 qr. 2 n. how many nails?
Ans. 262 nls
23. Reduce 44 hours 16 m. 8 sec. to seconds.
Ans. 159368 sec
24. How many inches in 4 yards 2 ft. 9 in.?
Ans. 177 in
25. Reduce 7 bush. 2 pks. 1 qt. 1 pt. to pints.
Ans. 483 pts
26. How many gills in 41 gals. 2 qts. 1 pt. 3 gills?
Ans. 1335 gls.
27. How many sec. in 4 days 18 hours 16 m. 45 sec.?
Ans. 411405 sec.
28. In 18 lbs. 6 oz. 19 pwt. 14 gr. how many grains?
Ans. 107030 gr.
29. In 2 cwt. 1 qr. 18 lbs. how many ounces?
Ans. 4320 oz.
30. In 3 miles 164 rods 12 ft. 9 in. how many inches?
Ans. 222705 in.
31. In 96 acres 120 rods, how many rods?
Ans. 15480 rods.
32. In 60 T. 20 ft. of round timber, how many ft.?
Ans. 2420 ft.
33. In 754 rods, how many acres?
Ans. 4 acres 114 rods.
34. In 4 years 48 days 10 h. 45 sec. how many sec.?
Ans. 130413645 sec.
35. In 90° 17′ 40″ how many sec.? Ans. 325060″.
36. In 67 yds. 2 qrs. 3 n. how many nails?
Ans. 1083 nails.
37. In 147 gals. 2 qts. 1 pt. 2 gls. how many gills?
Ans. 4726 gls.
38. In 471 h. how many seconds? Ans. 1695600 sec.

Sometimes, in performing a sum, we find a multiplier containing a denomination higher than that which is used as the integer in the supposition. In such cases, observe the following

RULE.

First, see what denomination you buy or sell by; and then see how many of that kind is contained in the multiplier.

APPLICATION.

1. At 6 cts. a quart, what will 2 pks. of walnuts cost?
Ans. 2 pks. = 16 qts. and 16 times 6 cts. = 96 cts.

2. At 14 cents a pound, what is the cost of 1 cwt. 2 qrs. of sugar? Ans. $23.52.

3. At 9 cents a pound, what cost 2 cwt. 3 qrs. 10 lbs. of sugar? Ans. $28.62.

4. If a man earns 27 dollars a month, how many dollars will he earn in 3 yrs. 6 m.? Ans. $1134.

5. A cannon ball goes at the rate of 9 miles a minute. How many miles would it go, if it continued at that rate for 3 hours? Ans. 1620 miles.

6. If the pendulum of a clock swing once in a second, how many times will it swing in a day? Ans. 86400 times.

7. At 10 cents a pound, what costs 1 cwt. 3 qrs. 8 lbs. of sugar? Ans. $20.40.

8. If 1 dollar is equal to 8s., what is $1024 equal to?
Ans. 409£. 12s.

9. If 1 dollar is equal to 6s., what is $355 equal to?
Ans. 106£. 10s.

10. If 1 dollar is equal to 7s. 6d. or 90d., what is $669 equal to? Ans. 250£. 17s. 6d.

11. If 1 dollar is equal to 4s. 6d., what is $285 equal to?
Ans. 64£. 2s. 6d.

12. If 1 dollar is equal to 4s. 8d., what is $361 equal to?
Ans. 84£. 4s. 8d.

To divide one denominate number by another of the same kind, observe the following

RULE.

First see what is the lowest denomination mentioned in both quantities, and reduce the divisor and the dividend to that denomination; then divide as in whole numbers.

APPLICATION.

1. How much sugar at 9d. a pound, can be bought for 5s. 6d.? ☞5s. 6d.=66d. Ans. $7\frac{3}{9}$ lbs.

2. How much beef at 6d. a pound, can be bought for 3£. 9s. 4d.? Ans. $138\frac{4}{9}$ lbs.

3. How much wheat at 8s. a bushel, can be bought for 5£. 4s.? Ans. 13 bush.

4. How much pork at 10d. a pound, can be bought for 3£. 5s. 9d.? Ans. $78\frac{9}{10}$ lbs.

5. How much cloth at 8s. a yard, can be bought for 8£. 5s.? Ans. $20\frac{5}{8}$ yds.

6. How many spoons, weighing 16 pwt. each, can be made out of 4 lbs. 7 oz. 4 pwt. of silver? Ans. 69 spoons.

7. How many coats, taking 1 yd. 3 qrs. 2 n. each, may be made of 35 yds. 2 qr. 2 n. of cloth? Ans. 19 coats.

8. How many bottles containing 1 pt. 3 gls. each, can be filled from a barrel of beer, guaging 30 gallons? Ans. $137\frac{1}{7}$ bottles.

9. How many ninepences are in 18£. 3s.? Ans. 484 ninepences.

10. How many guineas at 21s. each, are equal to 14£. 15s? Ans. $14\frac{1}{21}$ guineas.

11. How many dollars at 8s. are equal to 30£. 6s.? Ans. $\$75\frac{3}{8}$.

12. How many dollars at 7s. 6d. each, are equal to 9£. 4s. 6d.? Ans. $\$24\frac{54}{60}$.

13. How many dollars at 6s. are equal to 106£. 16s.? Ans. $\$356$.

REDUCTION OF DENOMINATE FRACTIONS TO VULGAR FRACTIONS.

THE pupil has already seen that one inch is $\frac{1}{12}$ part of a foot, because 12 inches make a foot. One ounce is $\frac{1}{16}$ part of a pound, because 16 oz. make a pound. One quart is $\frac{1}{4}$ part of a gallon. So is 1 pint the $\frac{1}{8}$ of a gallon, because 8 pints make a gallon. One gill is $\frac{1}{32}$ part of a gallon, because 32 gills make 1 gallon. This is called reducing a quantity of one denomination to the fraction of a higher denomination.

RULE.

Reduce the given quantity to the lowest denomination mentioned; and use the number thus obtained for the *numerator*. Then reduce to the same denomination, a unit of the higher denomination mentioned as the integer; and use that number as the *denominator*.

APPLICATION OF THE RULE.

1. What part of an hour is 1 minute? 7 minutes?

Ans. 1 *h.* = 60 *m.* *Therefore*, 1 *m.* = $\frac{1}{60}$ *of an hour.*

2. What part of a minute is 1 second? 14 seconds? 45 seconds?
3. What part of a gallon is 1 quart? 3 quarts?
4. What part of a quart is 1 pint?
5. What part of a pint is 3 gills?
6. What part of a day is 1 hour? 16 hours?
7. What part of a year is 1 day? 18 days? 47 days? 139 days?
8. What part of a foot is 1 inch? 6 inches? 11 inches?
9. What part of a yard is 2 feet?
10. What part of a mile is 127 rods? 273 rods?
11. What part of a yard is 3 quarters?
12. What part of a bushel is 1 peck? 3 pecks?

13. What part of a peck is 7 quarts? 3 quarts?

14. What part of a hundred weight is 97 lbs.? 108 lbs.?

15. What part of a quarter of a hundred weight is 8 lbs.? 16 lbs.?

16. What part of a shilling is 1 penny? What part of a penny is one farthing?

17. What part of a shilling is 1 farthing? 3 farthings?

18. What part of a gallon is 1 pint? 1 gill? 3 gills?

19. What part of an hour is 1 second? 6 seconds? 46 seconds?

20. What part of a day is 1 minute? 1 second? 34 seconds? Ans. 1 *d.* = 1440 *m.* *Therefore*, 1 *m.* = $\frac{1}{1440}$ *of a d.*

21. What part of a yard is 1 nail? 3 nails?

22. What part of a bushel is 1 quart? 1 pint?

23. What part of a ton is 1 quarter of a cwt.? 3 qrs.?

24. What part of a hundred is 1 ounce? 7 oz.? 15 oz.?

25. What part of a pound is 1 penny? 1 farthing?

26. What part of a gallon is 3 qts. 1 pt.? Ans. $\frac{7}{8}$.

27. What part of a day is 20 hours 30 minutes? Ans. $\frac{1230}{1440}$.

28. What part of a yd. is 2 qrs. 3 n.? Ans. $\frac{11}{16}$.

29. What part of a bushel is 3 pecks 5 qts. 1 pt.? Ans. $\frac{59}{64}$.

30. What part of a cwt. is 2 qrs. 25 lbs. 6 oz.? Ans. $\frac{1302}{1792}$.

31. What part of an oz. is 18 pwts. 23 grs.? Ans. $\frac{455}{480}$.

32. What part of an oz. is 3 dr. 1 sc. 18 grs.? Ans. $\frac{218}{480}$.

33. What part of a sq. yd. is 6 sq. ft. 94 sq. in.? Ans. $\frac{959}{1296}$.

34. What part of a degree is 35′ 48″? Ans. $\frac{2148}{3600}$.

35. What part of a £ is 4s. 7d.? Ans. $\frac{55}{240}$.

36. What part of a gallon is 2 qts. 1 pt. 3 gills? Ans. $\frac{23}{32}$.

37. What part of a yd. is 1 qr. 3 nls.? Ans. $\frac{7}{16}$.

38. What part of a day is 18 h. 43 m. 18 sec.?
Ans. $\frac{11233}{14400}$

39. What part of a ton is 18 C. 96 lbs. 14 oz.?
Ans. $\frac{607}{640}$.

40. What part of a ton is 14 cwt. 2 qrs. 27 lbs. 11 oz.?
Ans. $\frac{30427}{32000}$.

MISCELLANEOUS EXERCISES ON THE FOREGOING RULES.

1. How many times will a wheel 18 ft. 4 in. in circumference, turn round in going 150 miles? Ans. 43200 times.

2. How many sacks of 5 bush. 2 pks. may be filled out of 4 granaries which contain 64 bush. 2 pks. 4 qts. each? Ans. 47 sacks.

3. What is the value of 1 cwt. of sugar, at 8 cts. a pound? Ans. $8.96.

4. A company built 75 miles of road at the rate of $4.33 a rod. What was the whole expense? Ans. $103920.

5. Suppose the sun to be just 95000000 of miles from the earth; and a cannon ball, to move at the rate of a mile in 8 seconds. How long would it take the ball at that rate to reach the sun? Ans. 24 yrs. 36 d. 7 h. 6 m. 40 sec.

6. In £48 how many sets of shillings, nine-pences, six-pences, four-pences, and pence, having an equal number of each? Ans. 360.

7. How many rings each weighing, 5 pwt. 7 grs. may be made of 3 lb. 5 oz. 16 pwt. 2 grs. of gold? Ans. 158.

8. What will 5 cwt. 2 qrs. 18 lbs. of sugar come to at 12 cts. a pound? Ans. $76.08.

9. If in a journey a wheel 16 feet and 6 in in circumference turns round 13440 times, how many miles does it run? Ans. 42 miles.

10. A solar year is 365 days 5 h. 48 m. 48 sec. long. How many seconds does it contain? Ans. 31556928 sec.

11. What is the value of 1 cwt. of sugar, at 8d. per pound? Ans. £3, 14s. 8d.

12. If one cwt. of iron cost $3.72, what is the cost of 33 cwt. 1 qr. 22 lbs.? Ans. $124.42+.

13. At 9d. per quart, what will 16 gals. 2 qts. of molasses come to? Ans. £2, 9s. 6d.

14. How many cwt. of sugar at 12 cts. a pound may be bought for $76.08? Ans. 5 cwt. 2 qrs. 18 lbs.

15. In 25 guineas of 28s. each, how many dollars of 6s. each? Ans. $116.66⅔.

16. At £1, 2s. 6d. per yard, what will 20 yards of broadcloth cost in dollars of 8s.? Ans. $56.25.

17. A lady sent to a silversmith a tankard, that weighed 5 lbs. 3 oz.; and ordered it to be made into spoons, each of which was to weigh 2 oz. 2 pwt. How many spoons did it make? Ans. 30 spoons.

18. Sold 3 cwt. of tobacco, at 20 cts. a pound; how much is the amount? Ans. $67.20.

19. What is the value in dollars, of 672 yards of linen at 2s. 5d. per yard, Penn. currency? Ans. $216.53⅓.

20. If a man spend 8 cents a day, how much will he spend in a year? Ans. $29.22.

21. A goldsmith sold a tankard for £10, 12s. at the rate of 5s. 4d. per ounce. What was the weight of the tankard? Ans. 39 oz. 15 dwt.

22. What part of 18 cwt. 3 qrs. 17 lbs. 6 oz. is 12 cwt. 1 qr. 13 oz.? Ans. $\frac{21965}{43978}$.

23. What part of 67 days 19 hours 7 m. 19 s. is 41 d. 20 h. 14 sec.? Ans. $\frac{36144114}{95578350}$.

24. What part of 19° 14′ 42″ is 8° 34′ 29″? Ans. $\frac{308869}{692852}$.

25. What part of 7 yards 1 qr. 2 n. is 4 yards 2 qrs. 3 n.? Ans. $\frac{75}{118}$.

26. What part of 19s. 7½d. is £1, 4s. 6d.? Ans. $\frac{126}{157}$.

27. What part of 4 bush. 1 pk. 3 qts. 1 pt. is 14 bush. 6 quarts? Ans. $\frac{204}{1416}$.

28. What part of 97 C. 41 lbs. is 14 cwt. 1 qr. 27 lbs.?
Ans. $\frac{541}{3247}$.

29. What part of 36 cwt. 2 qrs. 18 lbs. is 36 C. 43 lbs.?
Ans. $\frac{3643}{4100}$.

30. What part of 14 yds. 3 qrs. is 1 yd.? And if 14 yds. 3 qrs. of cloth cost $24.50, what will a yard cost?
Ans. 1 yd. is the $\frac{4}{59}$ of 14 yds. 3 qrs.; and will cost 1.66\frac{6}{59}$.

31. What part of 18 gals. 2 qts. is 4 gals. 1 qt. 1 pt.? and if 18 gals. 2 qts. of wine cost 28 dollars, what is the cost of 4 gals. 1 qt. 1 pt.? Ans. $\frac{35}{148}$; and costs 6.62\frac{6}{37}$.

32. What part of 1 C. 75 lbs. is 1 C. 14 lbs.? And if 1 C. 75 lbs. of tea cost $200, what is the cost of 1 C. 14 lbs. of the same. Ans. $\frac{114}{175}$; and costs 130.28\frac{4}{7}$.

33. What part of 3 cwt. 1 qr. 12 lbs. is 1 cwt. 3 qrs. 8 lbs.? and if 3 cwt. 1 qr. 12 lbs. of tobacco cost $74.50, what is the cost of 1 cwt. 3 qrs. 8 lbs. of the same?
Ans. $\frac{51}{94}$; and 1 cwt. 3 qrs. 8 lbs. costs 40.42\frac{1}{47}$.

34. If 18 lbs. 9 oz. of coffee cost $3.00, what is the cost of 7 lbs. 14 oz.? Ans. 1.27\frac{3}{11}$.

35. If 42 gals. 1 qt. 1 pt. of brandy cost $37.00, what is the cost of 16 gals. 1 pt. 2 gls.? Ans. 14.13\frac{143}{330}$.

36. Four men hire a pasture for 25 dollars. A put in 28 cows; B, 31; C, 33; and D, 37. What ought each to pay? Ans. A, 5\frac{55}{129}$; B, 6\frac{1}{129}$; C, 6\frac{51}{129}$; D, 7\frac{22}{129}$.

37. A, B, and C, traded together. A put in $250; B, $350; and C, $300; and they gained $328. What was each man's share of the gain?
Ans. A, 91\frac{1}{9}$; B, 127$\frac{5}{9}$; C, 109\frac{1}{9}$.

ADDITION OF DENOMINATE FRACTIONS;

OR,

COMPOUND ADDITION.

INDUCTIVE EXERCISES FOR THE SLATE.

1. A GENTLEMAN has 6 kegs. One of them holds 4 gals. 2 qts. 1 pt.; another holds 4 gals. 1 qt. 1 pt.; another holds 5 gals. 2 qts.; another holds 3 gals. 3 qts. 1 pt.; another holds 3 gals. 2 qts. 1 pt.; and another holds 5 gals. 2 qts. 1 pt. How much do they all hold?

Ans. I add up the column of pints, which is equal to 5 pints, which = 2 qts. 1 pt. I add up the column of qts. which with 2 qts. from the pints = 14 qts. which = 3 gals. 2 qts. I add up the column of gallons, which with the 3 gals. from the quarts = 27 gallons.—Therefore the whole answer is

4 gal.	2 qts.	1 pt.
4	1	1
5	2	0
3	3	1
3	2	1
5	2	1
27	2	1

Example 2.

In this sum, when we add the column of feet, we find 67. But as 16½ feet make a rod, we say 33 *half feet* make a rod; and call the 67 feet 134 *half feet*. In 134 half ft. there are 4 times 33 half ft. and 2 half feet over. As these two half ft. equal 1 ft., we know that in 134 half ft. (that is, in 67 ft.) there is 4 rods and 1 foot over.

2 miles	200 r.	12 ft.	10 in.
4	180	11	8
3	300	12	9
6	296	11	0
8	310	12	10
9	267	6	6
36	277	1	7
34	76	5	3
36	277	1	7

In examples of this kind, if the number of remaining half feet is *odd*, see if the inches just placed in the answer are equal to another half foot. If they are not, put the odd half foot with them. If they are, transfer half a foot from the inches to the feet. Prove example 2 in this manner.

DEFINITION.

Compound Addition, or, as it is more properly called, *Addition of Denominate Fractions*, is the adding of two or more quantities that contain the denominational divisions of some measuring unit.

RULE.

First, State the question by putting one number under another, in such a manner that each denomination shall be directly under the same denomination in the last written number.

Second, Add the column of the least denomination, and see whether its sum will amount to one of the next higher denomination. If it does not, write it in the answer, under its own column.

Third, If the sum equals any in the next higher denomination, find by reduction how many it does equal. Carry that number to the next column, and write the remainder under the column added.

Fourth, Proceed in the same manner through all of the denominations.

APPLICATION.

[1st sum.]				[2d sum.]				
21 gals.	3 qts.	1 pt.	1 gl.	3 yrs.	47 d.	12 h.	30 m.	10 sec.
47	0	1	3	4	17	15	13	14
27	1	0	2	4	112	11	45	45
72	2	1	1	6	210	4	53	7
41	3	0	3	7	145	8	35	54
14	0	1	2	5	276	23	50	57

[3d sum.]

6 yds.	3 qrs.	2 n.
47	2	2
64	3	1
76	0	3
37	2	0
73	3	2

[4th sum.]

47 bush.	3 pks.	7 qts.	1 pt.
49	1	6	1
100	2	4	0
79	1	7	1
41	2	4	0
65	0	7	1

[5th sum.]

67 C.	20 lbs.	8 oz.
76	74	9
173	98	14
716	22	15
475	48	13
436	24	12

[6th sum.]

3 lbs.	6 oz.	18 pwt.	12 grs.
2	11	17	10
1	4	9	3
	11	19	20
1	10	17	23
	7	14	11

[7.]

1 lb.	6 ℥.	4 ʒ.	2 sc.	10 grs.
	11	7	1	18
	9	3	0	16
	7	6	1	19
1	4	4	0	8
1	6	7	0	12

[8.]

19 yrs.	300 d.	19 h.	45 m.	55 se.
17	325	14	54	16
8	232	18	16	41
12	106	14	18	9
6	174	12	30	30
18	337	22	49	34

[9.]

97 yds.	1 qr.	2 n.
46	0	0¼
14	3	3
47	2	0¼
34	1	3
27	0	1¼

[10.]

49 C.	24 lbs.	2½ oz.
62	92	4
21	75	12
42	48	14
18	89	9½
29	16	12½

[11.]

196 gals.	2 qts.	0 pt.	3 gl.
214	1	1	2¾
42	0	1	0
97	3	0	1¾
125	0	1	3
44	2	1	2¾

[12.]

86 bush.	2 pks.	1 qt.	1 pt.
74	3	2	1⅛
473	1	6	0⅜
423	3	4	1
765	2	7	0⅛
641	0	5	1⅜

[13.]

cwt.	qrs.	lbs.	oz.
18	3	27	6
42	2	25	8
27	2	8	12½
78	0	14	15
45	1	22	12½
52	3	27	10½

[14.]

a.	sq. r.	sq. ft.	sq. in.
10	100	6	100
6	140	18	124
4	120	200	140
7	155	217	96
4	45	171	121
6	124	216	112

[15.]

cords	sol. ft.	sol. in.
3	100	1947
4	124	1700
6	114	1647
5	122	1716
4	127	1626
6	105	1427

[16.]

deg.	′	″
48	40	30 1/11
69	35	45 10/11
94	53	54 11/11
49	13	12 1/11
46	17	42 9/11
64	45	47 11/11

[17.]

£.	s.	d.
4	12	9½
37	14	7
25	17	6½
14	8	7¾
6	10	4
18	12	9¾

[18.]

a.	sq. r.	sq. ft.
100	27	4
89	124	27
94	142	261
46	64	142
25	146	241
49	137	270

[19.]

£	s.	d.
327	5	9
84	10	10½
46	15	6½
427	0	8¾
91	2	7
4	8	11

[20.]

£	s.	d.
28	15	9
14	18	7
112	13	2½
126	11	9½
38	4	11
64	15	10½

21. A farmer raised in one field 40 bush. 1 pk. 6 qts. of corn; in another, 60 bush. 3 pks. 6 qts. of rye; in another, 94 bush. 2 pks. 4 qts. of oats; and in another, 204 bush. 1 pk. 2 qts. of wheat. How much grain did he raise?

Ans. 400 bush. 1 pk. 2 qts.

22. A lady purchased 6 yds. 2 qrs. 1 n. of cambric; 14 yds. 1 qr. 3 n. of sheeting; and 2 yds. 1 qr. 2 n. of linen. How many yards did she buy in all?

Ans. 23 yds. 1 qr. 2 n.

In some of the following sums, particular care must be given that each denomination be put in its proper place.

23. A merchant sold 4 hogsheads of molasses: one hogshead contained 75 gals. 2 qts. 1 pt.; another contained 84 gals. 1 pt.; another contained 87 gals. 1 qt.; and another 79 gals. How much did they all contain?

Ans. 326 gals.

24. A person traveled 6 days. The first day, he went 96 m. 67 rods; the second, 49 m. 94 r.; the third, 106 m. 300 r.; the fourth, 113 m. 260 r.; the fifth, 90 m. 100 r. 10 ft.; and the sixth, 44 m. 16 ft. How far did he travel in the six days? Ans. 500 m. 182 r. 9½ ft.

25. A farmer has 5 fields, containing as follows: 48 acres 100 r. 100 ft.; 36 a. 10 r. 97 ft.; 62 a. 140 r. 200 ft.; 84 a. 48 r. 160 ft.; 39 a. 120 r. 24 ft. How much in all?

Ans. 271 a. 100 r. 36½ ft.

26. A grocer bought 4 hogsheads of sugar, weighing as follows: No. 1, 9 C. 79 lbs.; No. 2, 8 C. 96 lbs.; No. 3, 7 C. 76 lbs.; No. 4, 8 C. 84 lbs.; and 2 barrels of sugar, weighing each 2 C. 71 lbs. What did the whole weigh?

Ans. 40 C. 77 lbs.

27. How many C. of sugar in 6 hogsheads, each 1142 lbs.? Ans. 68 C. 52 lbs.

28. What is the weight of 12 spoons, each weighing 3 oz. 2 pwt. 8 grs.? Ans. 37 oz. 8 pwt.

29. A silversmith sold a silver tea-pot, weighing 1 lb. 3 oz. 20 grs.; a silver bowl, weighing 1 lb. 1 oz. 14 pwt.; a cream-dish, weighing 11 oz. 14 grs.; a dozen forks, weighing 2 lb. 10 oz. 14 pwt. 6 grs.; and 6 silver spoons, weighing 1 lb. 3 oz. 16 grs. What is the weight of the whole? Ans. 7 lb. 5 oz. 10 pwt. 8 grs.

30. A druggist put together, of one kind, 4 drs. 2 sc. 8 grs.; of another kind, 3 drs. 1 sc. 14 grs.; of another, 2 sc. 18 grs.; of another, 3 drs. 14 grs.; and of another, 2 sc. 16 grs. How much did the whole mixture weigh?

Ans. 13 drs. 1 sc. 10 grs.

31. An ironmonger sold to one man 94 tons 6 C. of iron; to another, 47 T. 10 C. 14 lbs.; to another, 74 T. 97 lbs.; to another, 49 T. 14 C. 9 lbs.; to another, 22 T. 56 lbs.; to another, 12 T. 13 C. 40 lbs.; and to another, 13 C. 48 lbs. 8 oz. How much did he sell to them all?

Ans. 300 T. 18 C. 64 lbs. 8 oz.

32. A surveyor found that one side of a field measured 180 rods 10 feet 9 inches; another side, 99 r. 14 ft. 10 in.; another side, 146 r. 14 ft. 6 in.; and another side, 102 r. 9 in. How far is it around the field?

Ans. 529 r. 7 ft. 10 in.

33. How many square feet are there in three boards, containing each 96 feet 14 in.? Ans. 288 ft. 42 in.

34. A ship builder buys 5 lots of hewn timber, which measure 98 T. 26 ft., 49 T. 40 ft., 63 T. 27 ft., 35 T. 42 ft., 52 T. 36 ft. How much in the 5 lots?

Ans. 300 T. 21 ft.

35. How much round timber in 60 T. 30 ft., 504 T. 27 ft., 400 T. 25 ft., 94 T. 25 ft., 102 T. 31 ft., 89 T. 32 ft.?

Ans. 1253 T. 10 ft.

36. How much wine in 4 casks, containing 31 gals. 2 qts. 1 pt., 29 gals. 1 pt., 30 gals. 1 qt. 1 pt., 29 gals. 3 qts. 1 pt.? Ans. 121 gals.

SUBTRACTION OF DENOMINATE FRACTIONS;

OR,

COMPOUND SUBTRACTION.

INDUCTIVE EXERCISES FOR THE SLATE.

1. If you take 2 quarts from a gallon how much remains?

2. If you take 30 seconds from a minute, how much remains?

3. If you take 5 days from a week, how much remains?

4. If you take 4 quarts from a peck, what will remain?

5. If you take 1 peck from a bushel, what will remain?

6. If you take 6 ounces from a pound, what will remain?

7. If you take 100 rods from an acre, what will remain?

8. If you take 2 quarts from a gallon and 1 quart, what will remain?

9. If you take 2 ft. from 1 yd. 1 ft., what will remain?

10. If you take 9 inches from 1 foot 6 inches, what will remain?

11. If you take 3 quarters from 1 yard 2 qrs., what will remain?

12. If you take 2 pecks from 1 bushel 1 peck, what will remain?

13. If you take 12 ounces from 1 lb. 6 oz., what will remain?

14. If you take 20 grains from 1 pwt. 16 grs., what will remain?

If you wish to subtract 20 grains from 1 pwt. 22 grains, you would immediately say the answer is 1 pwt. 2 grains. But when you subtract 20 grains from 1 pwt. 16 grs., you *first* reduce the 1 pwt. to grains, and add in the 16 grs.; and *afterwards* subtract. This is called *borrowing the* 1 *pwt.* The next sum is performed on the same principle.

15. From 2 feet 3 inches subtract 10 inches.

Here, as 10 in. cannot be taken from 3 in., we borrow 1 ft. to be put with the 3 in. We then have 15 in. from which to subtract the 10 in. The remainder is 5 in.; and as we borrowed 1 of the feet, we have but 1 ft. left, to complete the answer.

2 ft.	3 in.
	10
1	5 Ans.

16. From 3 yds. 1 qr. subtract 1 yd. 3 qrs.

In this sum we borrow 1 yd. to be put with the 1 qr. We have then 5 qrs., from which we subtract the 3 qrs. We may then consider, either that we subtract the 1 yd. from the two remaining yds., or that we increase the 1 yd. to 2, and subtract the 2 yds. from the 3 yards. (See page 44.)

3 yds.	1 qr.
1	3
1	2 Ans.

17. From 8 lbs. 6 oz. subtract 5 lbs. 12 oz.

☞ Instead of adding the 16 oz. and 6 oz. together, and then subtracting the 12 oz.; we may obtain the same answer by saying

12 *oz. from* 16 *oz. leave* 4 *oz., and the* 6 *oz. added in equals* 10 *oz. &c.*

8 lbs.	6 oz.
5	12
2	10 Ans.

18. From £9, 5s. subtract £7, 13s.

19. From 7 bush. 2 pks. 1 qt., subtract 4 bush. 1 pk. 5 quarts.

20. From 10 days 6 h. 40 min. 15 sec., subtract 6 d. 3 h. 45 min. 30 sec.

21. From 6 gals. 2 qts. 0 pt., subtract 2 gal. 1 qt. 1 pt.

22. From £2, subtract 3 qrs.

In this sum, as there are no qrs. to be subtracted from, it is necessary to borrow a penny from a higher denomination. 1d. taken from £2, leaves £1, 19s. 11d., as in the margin. From this 1d., which equals 4 qrs., subtract the 3 qrs. Then the rest of the sum is performed as before.

£2,	0s.	0d.	0	qrs.
			3	
	or,			
£1,	19s.	11d.	4	qrs.
			3	
1	19	11	1	

DEFINITION.

Compound Subtraction, or *Subtraction of Denominate Fractions*, teaches how to find the difference between two numbers that are composed of denominational divisions.

Rule.

First, State the question, so that each denomination in the lesser number shall be under the same denomination in the greater.

Second, Begin with the smallest denomination, and subtract the under number from the one above it. But if the under number be the greatest, take it from as many as it takes of that denomination to make 1 in the next greater; and to that result add the upper number, and set down the sum. Then, before you subtract in the next denomination, either add one to the under number, or consider the upper number as *one* less.

APPLICATION.

[1.]					[2.]				
From	18	gals.	1	qt. 1 pt.	3	gls.	47 d.	14 h.	30 m. 30 sec.
Take	9		3	0	4		39	20	15 45
Ans.									
Proof.									

[3.]

96 yds. 2 qrs. 1 n.
48 3 2

[4.]

44 bu. 2 pks. 1 qt.
36 3 2

[5.]

16 C. 1 qr. 12 lbs.
8 2 16

[6.]

44 yrs. 200 d. 20 h. 15 m. 2 sec.
23 300 20 12 35

[7.]

94 miles 40 rds. 14 ft. 6 in.
49 252 16 1

[8.]

9 lbs. 1 oz. 19 pwt. 14 grs.
6 11 17 23

[9.]

5 lbs. 7 oz. 6 dr. 1 scr. 10 grs.
4 11 7 2 18

[10.]

18 T. 14 C. 3 qrs. 20 lbs. 12 oz.
14 16 1 22 14

[11.]

1 rod 47 sq. ft.
142

[12.]

16 T. 14 cwt. 2 qrs. 26 lbs. 10 oz.
8 16 3 27 12

[13.]

$196\frac{3}{4}$ miles.
$137\frac{5}{14}$

[14.]

97 miles 200 rods 14 ft. 7 in.
79 300 16 9

[15.]

44 rods 5 yds. 2 ft. 6 in.
12 4 2 9

[16.]

4 weeks 6 d. $20\frac{1}{2}$ h.
3 4 22

[17.]

14 yds. 2 ft. $6\frac{3}{4}$ in.
7 1 $9\frac{1}{4}$

[18.]

6 yds. 2 qrs. 1 n.
2 3 3

[19.]

10 bush. 1 pk. 6 qts. 0 pt.
9 2 7 1

[20.]

3 T. 4 C. 20 lbs. 9 oz.
1 6 22 12

[21.]

1 lb. 6 oz. 10 pwt. 12 grs.
9 14 14

[22.]

4 drs. 1 scr. 16 grs.
3 2 12

[23.]

50 sol. ft. 1427 sol. in.
27 1600

[24.]

300 acres 100 sq. rods. 100 sq. ft.
276 145 200

25. From 96 yds. 1 qr. take 14 yds. 2 qrs. 2 n.
Ans. 81 yds. 2 qrs. 2 ns.

26. From a piece of cloth containing 35 yds. there were taken three suits, each requiring 3 yds. 1 qr. How much was left?
Ans. 25 yds. 1 qr.

27. From 125 bush. 2 pks. 1 qt. take 49 bush. 3 pks. 7 qts.
Ans. 75 bush. 2 pks. 2 qts.

28. A certain cask contained 109 gals. 1 qt. There was sold 74 gals. 3 qts. How much remains in the cask?
Ans. 34 gals. 2 qts.

29. A man started on a journey of 425 miles, and has gone 300 miles 200 rods 10 ft. How far has he to go?
Ans. 124 m. 119 rds. 6½ ft.

30. From 114 yds. 1 ft. 6 in. take 100 yds. 1 ft. 1 in.
Ans. 14 yds. 5 in.

31. From 94 gals. 2 qts. take 41 gals. 1 pt.
Ans. 53 gals. 1 qt. 1 pt.

32. From 42 yds. 2 n. take 29 yds. 1 qr. 3 n.
Ans. 12 yds, 2 qrs, 3 n.

33. A farmer owned 120 acres 70 rods, but had paid for only 74 acres 100 rods. How much was unpaid for?
Ans. 45 acres 130 rods.

34. A man bought 36 cwt. 1 qr. 16 lbs. of hay; but afterwards sold 22 cwt. 20 lbs. How much did he retain?
Ans. 14 cwt. 24 lbs.

35. A merchant having 14 C. 2 lbs. of sugar, sold 8 C. 47 lbs. How much had he left? Ans. 5 C. 55 lbs.

36. A goldsmith having 2 lbs. 6 oz. 12 pwt. of silver, made from it 6 spoons, weighing 1 lb. 3 oz. 13 pwt. 12 grs. How much silver was there left?
Ans. 1 lb. 2 oz. 18 pwts. 12 grs.

37. John is 10 years 2 mo. 6 days old; and Henry is 14 years 9 mo. 20 days old. Who is the oldest, and how much? Ans. Henry is 4 y. 7 m. 14 d. older than John.

38. From 1832 years 4 mo. 10 days, take 1814 years 9 mo. 18 d. 16 hours. Ans. 17 y. 6 m. 21 d. 8 h.

NOTE.—To find the length of time from one date to another; set down the number of the year, the number of months that *have past*, and the number of days in the month that has not passed. The later year must be put above the earlier.

39. How long is it from A. D. 1804, March 28th, to July 4th, 1832.

1832	yrs.	6 m.	4d.
1804	2		28
Ans. 28	3		6

40. How long is it from A. D. 1836, June 25th, to 1839, May 2d? Ans. 2 yrs. 10 m. 7 d.

41. How long is it from Jan. 8th, 1827, to Dec. 7th, 1839? Ans. 12 yrs. 10 m. 29 d.

42. How long is it from Nov. 16th, 1838, to May 9th, 1840? Ans. 1 yr. 5 m. 23 days.

MULTIPLICATION OF DENOMINATE FRACTIONS;

OR,

COMPOUND MULTIPLICATION.

DEFINITION.

COMPOUND MULTIPLICATION, or *Multiplication of Denominate Fractions*, teaches to multiply a number that contains a denominational division of a measuring unit.

RULE.

First, State the question as in simple multiplication.

Second, Begin at the right hand, and multiply each denomination by itself, taking care to carry as in compound addition.

APPLICATION.

[1.] Multiply 9 gals. 1 qt. 1 pt. 2 gls. By 3

[2.] 14 days 8 h. 10 m. 42 s. 7

[3.] 18 miles 40 r. 15 ft. 9 in. 6

[4.] 6 yds. 2 qr. 1 n. 8

[5.] 9 bush. 2 pks. 6 qts. 1 pt. 5

[6.] 9 C. 29 lbs. 6 oz. 4

[7.]
14 cwt. 1 qr. 27 lbs. 9 oz.
7

[8.]
8 years, 104 days, 14 h. 8 m.
9

[9.]
8 yds. 2 ft. 6 in.
8

[10.]
1 lb. 6 oz. 18 pwt. 22 grs.
6

[11.]
18 sq. a. 150 sq. rods, 100 sq. ft. 47 sq. in.
9

12. If a merchant sells 7 hogsheads of sugar, each weighing 7 C. 2 qr. 8 lbs. 9 oz., what is the weight of the whole? Ans. 53 C. 9 lbs. 15 oz.

13. If one side of a field measures 80 rods 14 ft. 9 in., and the three other sides have the same length, what length of fence will it take to inclose the field?
Ans. 323 r. 9 ft. 6 in.

14. How much wool is there in 6 packs, each weighing 1 cwt. 3 qrs. 19 lbs.? Ans. 11 cwt. 2 qrs. 2 lbs.

15. A man has a cask of wine containing 103 gals. 2 qts. 1 pt. After filling 4 kegs, each containing 18 gals. 1 pt. 2 gls., how much remains in the cask?
Ans. 30 gals. 3 qts. 1 pt.

16. A merchant bought 300 bars of iron, weighing each 2 qrs. 8 lbs. He afterwards sold 4 parcels of 33 cwt. 9 lbs. 8 oz. each. How much remained?
Ans. 39 cwt. 10 lbs.

17. From a piece of cloth containing 35 yds. 2 qrs. 1 n., a tailor cut 4 suits, requiring 6 yds. 3 qrs. 2 n. each. How much cloth remained? Ans. 8 yds. 1 n.

18. A merchant sold 8 yds. of cloth at 49s. 2½d. per yd. How much did the whole come to? Ans. 19£. 13s. 8d.

19. What is the value of 5 cwt. of indigo, at 8s. 11½d. per lb.? [Do not reduce the *price.*] Ans. £250, 16s. 8d.

20. What is the weight of 7 tierces of rice, each weighing 5 cwt. 2 qrs. 16 lbs.? Ans. 39 cwt. 2 qrs.

21. In 8 casks of corn, each containing 2 bush. 3½ pks., how much corn have I? Ans. 23 bush.

22. What is the cost of 108 lbs. of tea at 8s. 5½d. per lb.? Ans. £45, 13s. 6d.

23. What is the cost of 5 cwt. of iron at 3¼ pence per pound? Ans. £8, 15s.

24. What is the weight of 15 barrels of sugar, if each weighs 3 cwt. 1 qr. 14 lbs.? Ans. 50 cwt. 2 qrs. 14 lbs.

25. What is the cost of 83 boxes of indigo at £1, 19s. 10½d. per box? Ans. £165, 7s. 10½d.

26. What is the cost of 127 lbs. of tea at 8s. 7d. per pound? Ans. £54, 10s. 1d.

27. How much molasses is contained in 25 hhds.; each hhd. guaging 61 gal. 1 qt. 1 pt? Ans. 1534 gal. 1 qt. 1 pt.

28. What is the weight of 42 tierces of rice, each weighing 5 cwt. 2 qrs. 16 lb.? Ans. 237 cwt.

29. How many yards of cloth in 36 pieces, each piece containing 25 yds. 3 qrs.? Ans. 927 yds.

30. What is the weight of 36 silver spoons, each weighing 3 oz. 6 pwt.? Ans. 9 lb. 10 oz. 16 pwt.

31. If a vessel sails at the rate of 2° 34′ 16″ a day for 17 days together, what will be her whole distance?
Ans. 43° 42′ 32″.

DIVISION OF DENOMINATE FRACTIONS;

OR,

COMPOUND DIVISION.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. WHAT is $\frac{1}{2}$ of 6 ft. and $\frac{3}{12}$ of a foot? [As on page 111.]
2. What is $\frac{1}{3}$ of 9 ft. and $\frac{6}{12}$ of a foot?
3. What is $\frac{1}{4}$ of 9 ft. and $\frac{9}{12}$ of a foot?
4. What is $\frac{1}{3}$ of 7 lbs. and $\frac{8}{16}$ of a pound?
5. What is $\frac{1}{6}$ of 9s. and $\frac{6}{12}$ of a shilling?
6. What is $\frac{1}{7}$ of 10 yds. and $\frac{2}{4}$ of a yard?
7. What is $\frac{1}{2}$ of 18 pks. and $\frac{1}{8}$ of a peck?
8. How many feet and inches is in $\frac{1}{7}$ of a pole that is 37 feet long? Ans. $\frac{1}{7}$ *of* 35 *feet is* 7 *feet; and the other* 2 *feet equals* 24 *inches.* $\frac{1}{7}$ *of* 24 *inches is* $4\frac{4}{7}$ *inches. Therefore* $\frac{1}{7}$ *of* 37 *feet, is* 7 *feet* $4\frac{4}{7}$ *inches.*
9. How many feet and inches is $\frac{1}{6}$ of 47 feet? 51 feet? 62 feet? 67 feet?
10. How many feet and inches is $\frac{1}{9}$ of 13 feet? 29 feet? 41 feet? 73 feet? 87 feet? 98 feet?
11. How many bushels and pecks are there in $\frac{1}{2}$ of 5 bushels? $\frac{1}{3}$ of 7 bushels? $\frac{1}{4}$ of 6 bushels 1 peck? $\frac{1}{5}$ of 7 bushels 3 pecks?
12. How much is $\frac{1}{3}$ of 5 bushels 3 pecks? $\frac{1}{4}$ of 7 bushels 2 pecks? $\frac{1}{2}$ of 9 bushels 3 pecks? $\frac{1}{8}$ of 10 bushels 1 peck?

13. How many gallons and quarts are in $\frac{1}{3}$ of 7 gallons? $\frac{1}{4}$ of 10 gallons? $\frac{1}{5}$ of 9 gallons 2 quarts? $\frac{1}{6}$ of 10 gallons 3 quarts?

14. How many gallons and quarts are in $\frac{1}{4}$ of 6 gallons 1 quart? $\frac{1}{7}$ of 7 gallons 3 quarts? $\frac{1}{7}$ of 10 gallons 1 quart? $\frac{1}{8}$ of 14 gallons 3 quarts?

15. How many gallons and quarts are in $\frac{1}{7}$ of 23 gallons 2 quarts? $\frac{1}{8}$ of 31 gallons 1 quart? $\frac{1}{9}$ of 46 gallons 1 quart? $\frac{1}{10}$ of 37 gallons 3 quarts?

16. How many bushels and pecks are in $\frac{1}{7}$ of 32 bushels 1 peck? $\frac{1}{8}$ of 37 bushels 3 pecks? $\frac{1}{9}$ of 41 bushels 2 pecks? $\frac{1}{10}$ of 32 bushels 1 peck?

17. How many pounds and ounces are there in $\frac{1}{3}$ of 7 pounds? $\frac{1}{5}$ of 8 pounds? $\frac{1}{3}$ of 11 pounds? $\frac{1}{4}$ of 13 pounds?

18. How many pounds and ounces are there in $\frac{1}{2}$ of 5 pounds and 8 ounces? $\frac{1}{3}$ of 7 pounds and 5 ounces? $\frac{1}{4}$ of 9 pounds and 10 ounces.

19. How many pounds and ounces are there in $\frac{1}{4}$ of 5 pounds and 12 ounces? $\frac{1}{5}$ of 12 pounds and 14 ounces?

INDUCTIVE EXERCISES FOR THE SLATE.

The preceding exercises for mental Arithmetic are well calculated for this purpose. The pupil should perform them on his slate.

DEFINITION.

Compound Division is the division of quantities that are composed of denominate numbers. It is always fractional.

RULE.

First, State the question as in simple division.

Second, Divide the left hand denomination; and, if there is any remainder, reduce that remainder to the next lowest

denomination, and add the number in that denomination, as in reduction.

Third, Divide the sum now found, and proceed, as before, to the next denomination; and so on, till all is divided.

Example 1.

Divide £1, 17s. 10½d. by 3.

Ans. £1, 17*s.* = 37*s.* ⅓ *of* 37*s.* = 12*s.* *and* 1*s. over.* 1*s.* 10*d.* = 22*d.* ⅓ *of* 22*d.* = 7*d. and* 1*d. over.* 1½*d.* = 6 *farthings*, ¼ *of* 6 *q s.* = 2 *qrs. or* ½*d.*

3)£1, 17s. 10½d.
12. 7½

Example 2.

Divide £68, 5s. 7¾d. by 19.

£ s. d. £ s. d.

19)68 5 7¾(3 11 10½
57
11
20 11 times 20s. + 5s. =
19)225(11s.
209
16
12 16 times 12d. + 7d. =
19)199(10d.
190
9
4 9 times 4 qrs. + 3 qrs. =
19)39(2 qrs.
38
1 remainder.

The answer is £3, 11s. 10½d. ¼ 1.

3. Divide £6, 9s. 8½d. by 6½.

	£	s.	d.			
6½)	6	9	8½			
2			2	£	s.	d.
13)12	19		5½	(0	19	11½ Ans.

APPLICATION.

1. Divide 77 gals. 2 qts. 1 pt. 1 gl. by 4.
Ans. 19 gal. 1 qt. 1 pt. 1¼ gl.

2. Divide 44 years 19 h. 40 m. 16 sec. by 9.
Ans. 4 yrs. 324 ds. 12 h. 51 m. 8⅑ sec.

3. Divide 89 miles 300 r. by 7.
Ans. 12 m. 271 r. 7 ft. $\frac{6}{7}$ in.

4. Divide 44 feet, 9 in. by 7. Ans. 6 ft. 4$\frac{5}{7}$ in.

5. Divide 62 yds. 1 qr. 3 n. by 6.
Ans. 10 yds. 1 qr. 2⅜ n.

6. Divide 94 bush. 1 pk. 7 qts. 1 pt. by 8.
Ans. 11 bush. 3 pks. 1 qt. 1$\frac{7}{8}$ pt.

7. Divide 49 C. 49 lbs. 8 oz. by 14.
Ans. 3 C. 53 lbs. 8$\frac{8}{14}$ oz.

8. Divide 62 cwt. 1 qr. 24 lbs. 12 oz. by 16.
Ans. 3 cwt. 3 qr. 17 lbs. 4$\frac{12}{16}$ oz.

9. Divide 6 oz. 14 pwt. 19 gr. by 6.
Ans. 1 oz. 2 pwt. 11$\frac{1}{6}$ grs.

10. Divide 9 oz. 4 dr. 1 sc. 16 grs. by 12.
Ans. 6 dr. 1 sc. 3 gr.

11. Divide 80 acres 90 r. by 21. Ans. 3 a. 133$\frac{17}{21}$ r.

12. Divide 44 deg. 30′ 29″ by 9. Ans. 4° 56′ 43$\frac{3}{9}$″.

13. If 8 yds. of cloth cost 19£. 13s. 8d., how much is it per yard? Ans. 2£. 9s. 2½d.

14. If 1 doz. silver spoons weigh 39 oz. 12 pwt., what is the weight of each? Ans. 3 oz. 6 pwt.

15. Suppose a man traveled 122 miles 180 rods in 5 days, how far did he travel each day, on an average?
Ans. 24 miles 164 rods.

16. If 17 yds. of cloth cost 47£. 4s. 11d., how much is it per yd.? Ans. 2£. 15s. 7d.

17. Divide 14 gallons 3 qts. among 5 men.
Ans. 2 gallons 3 qts. 1 pt. $2\frac{2}{5}$ gls.

18. If 5 equal loads of hay weigh 4 T. 16 cwt. 18 lbs., what does 1 load weigh? Ans. 19 cwt. 26 lbs.

19. A gentleman divided 742 a. 124 r. of land equally between his 5 sons. How much had each?
Ans. 148 a. $88\frac{4}{5}$ rods.

20. How far must a man travel a day to go 642 miles in 7 days? Ans. 91 m. 228 r. 9 ft. $5\frac{1}{7}$ in.

21. A piece of land, consisting of 46 acres, produced 2000 bushels of corn. What was the average produce of 1 acre? Ans. 43 bush. 1 pk. $7\frac{14}{16}$ qts.

22. A person having died, left a farm of 362 acres 94 r., of which his widow was to have $\frac{1}{3}$, and the remainder to be divided equally among 3 children. What was the share of the widow and of each child? Ans. Widow's share 120 a. 138 r.; each child's share 80 a. 92 r.

23. If $10\frac{1}{4}$ yards of cloth cost £24, 12s.; how much is it per yard? Ans. £2, 8s.

24. Divide 13 lb. $7\frac{2}{3}$. $2\frac{2}{3}$. $1\frac{9}{3}$. 4 gr. by 4.
Ans. 3 lb. $4\frac{2}{3}$. $6\frac{2}{3}$. $1\frac{9}{3}$. 16 gr.

25. If 8 bags contain 23 bushels of wheat, how much is there in each bag? Ans. 2 bush. 3 pk. 4 qts.

26. If $8\frac{3}{4}$ yds. of cloth cost 7s. 7d. $3\frac{1}{2}$qrs.; how much is it per yard? [Reduce the yards to $\frac{1}{4}$ths.] Ans. $10\frac{1}{2}$d.

27. If 9 men are to work 114 hours, 45 minutes on a job of work, what is each man's share of the time, supposing their shares to be equal? Ans. 12 h. 45 m.

28. A prize was taken worth £963, 18s. To the captain is given $\frac{1}{3}$ of it, and the remainder is divided equally among 6 sailors. What is the share of each?
Ans. Captain's, £321, 6s.; each sailor's, £107, 2s.

29. If 9 tons of hay cost £41, 3s. 6d. what will 1 ton cost? Ans. £4, 11s. 6d.

30. A piece of land containing 96 acres, 2 roods, 16 rods, is to be divided equally among seven brothers. How much land is to be given to each? Ans. 13 a. 3 roods, 8 rods.

31. If 8 lb. 3 oz. 15 pwt. 18 grs. of gold be coined into 380 guineas, what is the weight of each? Ans. 5 pwt. 6 grs.

32. If you divide 9 ton, 3 cwt. 16 lb. into 8 equal parts, how much will be in each part? Ans. 1 T. 2 cwt. 3 qr. 16 lb.

33. A merchant wishes to divide 7 T. 16 cwt. 3 qr. 21 lb. equally into 63 barrels. How much will there be in each barrel? Ans. 2 cwt. 1 qr. 27 lb.

34. 52 pails of beer made 175 gal. 2 qts. How much did the pail contain? Ans. 3 gal. 1 qt. 1 pt.

35. 27 men own a front of 1 mile. How much is each man's share of the fence. Ans. 11 rods, 14 ft. $\frac{2}{3}$ in.

APPLICATION OF THE FOREGOING RULES.

1. Two persons depart from the same place at the same time: one travels 30, the other 35 miles a day. How far are they distant at the end of 7 days, if they travel both the same road; and how far if they travel in contrary directions? Ans. 35, and 455 miles.

2. George Washington was born on 22d February, 1732 How long from that time till February 22d, 1836?
Ans. 104 years.

3. A gentleman bought a hundred of sugar, at 7 cts. a pound. What was the cost of it? Ans. $7.

4. What is $\frac{1}{3}$ of a man's property that is valued at 27543 dollars? Ans. 9181 dollars.

5. A person's income is 2476 dollars a year. What is that a day? Ans. $6.78\frac{2}{3}$.

6. A gentleman gave by his will $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his son, and $\frac{1}{5}$ to his daughter, and the remaining part to charitable purposes. How much did he give to each, supposing him to be worth $153742?

Ans. To the wife, 51247.33\frac{1}{3}$; to the son, $38435.50; to the daughter, $30748.40; and in charity, 33310.76\frac{2}{3}$.

7. A merchant bought 402 yds. of cloth, at $2.75 a yd., and sold it at $2.97 a yd. What did he gain on the whole? Ans. $88.44.

8. A merchant bought 425 yds. of calico, at 12$\frac{1}{2}$ cts. a yard; and sold it at 15 cts. a yd. What did he gain on the whole? Ans. 10.62\frac{1}{2}$.

9. A grocer bought 236 lbs. of tea, at 1.16\frac{1}{2}$ a pound; but, as he was deceived in its quality, he sold it at 87$\frac{1}{2}$ cts a lb. What did he lose on the whole? Ans. $68.44.

10. A grocer bought 6 C. of sugar, at $8.25 a hundred, and sold it at 12 cts. a pound. What was his gain? Ans. $22.50

11. A merchant bought 210 gals. of wine, at $1.75 a gal. What must be his price a gallon to gain 50 dollars on the whole? Ans. 1.98\frac{17}{21}$.

12. A merchant bought 3 pieces of cloth, measuring each 23 yards, at $5.37 a yard. At what rate must he sell it a yard, to gain $62.50 on the whole? Ans. 6.27\frac{19}{69}$.

13. A dry goods merchant bought 100 yards of muslin, for $155.50. At what rate must he retail it to gain $37 on the whole? Ans. 1.92\frac{1}{2}$.

14. A merchant bought 1825 lbs. of coffee, at 12$\frac{1}{2}$ cts. a pound. At what rate must he sell it a pound to gain $18.25? Ans. 13$\frac{1}{2}$ cts.

15. A grocer sold 1400 lbs. of sugar for 9 cts. a pound, and by so doing gained $20. What did it cost him a pound? Ans. 7$\frac{4}{7}$ cts.

16. A flour merchant sold 60 barrels of flour for $6.12\frac{1}{2}$ a barrel, and gained $20. What did each barrel cost him?

Ans. $5.79\frac{1}{6}$.

17. A merchant has 3 C. 28 lbs. of tobacco, which he values at 16 cts. a pound. What is it all worth; and how much tea, at 97 cts. a pound, will pay for it?

Ans. Whole worth, $52.48; it will purchase $54\frac{10}{97}$ lbs. of tea.

18. How much tea, at $1.37\frac{1}{2}$ a lb., will pay for 3 C. of sugar at 9 cts. a pound? Ans. $19\frac{7}{11}$ lbs.

19. How many yards of muslin, at 18 cts. a yard, must be given for 74 yards of linen, worth 47 cts. a yard?

Ans. $193\frac{2}{3}$ yds.

20. How many yards of broadcloth at $5.25 cts. a yard, will pay for 123 yds. of silk, at $62\frac{1}{2}$ cts. a yd.?

Ans. $14\frac{6}{14}$ yds.

21. Two merchants wish to barter. One has coffee, which he sells for 22 cts. a pound: the other has sugar, that he sells for cash at $8 a C.; but in barter he asks $8.50. How many pounds of coffee will pay for 634 lbs. of the sugar? Ans. $244\frac{21}{22}$ lbs.

22. What cost 34 yds. of cloth, at 4 s. 6 d. a yd.?

Ans. 7£. 13 s.

23. A man bought 5 bales of cotton. The first weighed 4 cwt. 1 qr. 13 lb.; the second weighed 3 cwt. 2 qrs. 18 lb.; the third, 4 cwt. 2 qrs.; the fourth and fifth each 4 cwt. 23 lbs. What was the weight of the whole?

Ans. 20 cwt. 3 qrs. 19 lbs.

24. A countryman sold to a merchant a load of wood for $4.37\frac{1}{2}$; and received 8 lbs. of sugar, at 11 cents a pound; $\frac{1}{2}$ pound of tea, at 75 cts. a pound; 1 gallon of molasses, at 30 cts.; and then told the merchant if he would give him a bill, stating how much was not paid for, he would trade it out at another time. How much should the bill be?

Ans. $2.82.

25. A countryman sold 40 lbs. of butter, at 16 cts. a pound; 4 doz. of eggs, at 12½ cts. a doz.; and a bushel of peas, worth $1.25; and received goods in pay to the amount of $4.49; and a due bill for the balance. How much was the due bill? Ans. $3.66.

26. The revolutionary war commenced April 19th, 1775, and lasted till Jan. 20th, 1783. How long did it continue?
Ans. 7 years 9 mo. 1 day.

27. The last war between England and the United States continued 2 years 8 months 18 days. How much longer was the revolutionary war than the last?
Ans. 5 years 13 d.

28. At $12.50 per ton, what is the cost of 1 cwt. of hay? of 3 cwt.? of 14 cwt. Ans. 62½ cts., $1.87½, $8.75.

29. At $14 per C. what would 3 C. of sugar come to?
Ans. $42.

30. A merchant bought 24 barrels of flour for $140, and sold it so as to gain 75 cts. on a barrel. At what did he sell it a barrel? Ans. 6.58\frac{1}{2}$.

31. A merchant bought a hogshead of wine, containing 76 gallons, for $80. At how much must he sell it a gallon to gain 14 dollars? Ans. 1.23\frac{13}{19}$.

32. A merchant bought 1 cwt. of tobacco for $10. At how much per lb. must he sell it to gain $2.32? Ans. 11 cts.

33. A merchant sold 266 yds. of cotton cloth, at 16 cts. a yard; and received 5 cwt. of sugar as pay. What did the sugar cost a pound? Ans. 7$\frac{3}{5}$ cts.

34. A tallow chandler contracted to furnish 27 T. 14 cwt. 1 qr. 16 lbs. of tallow for $4800. What did it cost a pound; and, at that rate, what did it cost a cwt.; and what, a ton?
Ans. 7$\frac{11}{15}\frac{13}{52}\frac{39}{23}$ cts. a pound, and 8.65\frac{1}{15}\frac{2}{52}\frac{60}{23}$ a cwt., and 173.16\frac{37}{15}\frac{32}{52}\frac{}{23}$ a ton.

35. What is the cost of 4 chests of tea, each weighing 2 cwt. 3 qrs. 21 lbs. at $1.14 a pound? Ans. $1500.24.

36. At $10 per cwt. what is the cost of 1 lb. of sugar?
Ans. $8\frac{3}{4}$cts.

37. How many bottles, holding 3 pints each, may be filled with a barrel of cider containing 34 gallons?
Ans. 90$\frac{2}{3}$ bottles.

38. How many suits of clothes, taking 3 yds. 1 qr. 2 n. each, may be cut from a piece measuring 50 yds. 2 qrs. 2 n.?
Ans. 15 suits.

39. At 17 cts. a pound, what is the cost of 3 cwt. of chocolate? Ans. $57.12.

40. At $7 a ton, what is the cost of a bushel of coal? 1 ton = 20 bushels. Ans. 35 cts.

41. A man gave 325 dollars for 1 acre of land. What was that a rod? Ans. 2.03\frac{1}{2}$.

42. A gentleman purchased a building lot that was 15 ft. wide and 52 ft. deep, at the rate of 75 cts. a sq. ft. What was the cost of the lot? and what would be the cost of an acre at the same rate?
Ans. The lot cost $585; and an acre would cost $32670.

43. Four men purchased a lot of ground measuring 678 acres 90 rods. How much was the share of each?
Ans. 169 a. 102$\frac{1}{2}$ r.

44. A grocer purchased 14 cwt. 1 qr. of sugar. How many pounds in the whole? Ans. 1596 lbs.

45. A wine dealer wishes to fill an equal number of 3 sorts of bottles from a cask containing 94 gals. 1 pt. 3 gls. The bottles contained 3 gills, 1 pint, and 1 quart. How many sets of bottles did it take? Ans. 201 sets.

46. At 7 s. 6 d. a yd., how many yds. can I buy for 20£. 14 s. 6 d.? Ans. 55 yds. 1$\frac{1}{15}$ qr.

47. If $\frac{1}{7}$ of a dollar will pay for 1 yd., what will purchase 90 yds.? Ans. 12$\frac{6}{7}$ dollars.

48. If a ship was sold for £1689, 15s. 4d., what was the value of $\frac{5}{8}$ of her? Ans. £1056, 2s. 1d.

49. A merchant bought 112 bars of iron, weighing 56 cwt. 1 qr. 11 lb. of which he has sold 59 bars weighing 29 cwt. 3 qrs. 21 lb. How many bars has he left, and what is the weight? Ans. 53 bars, weighing 26 cwt. 1 qr. 18 lb.

50. What is the weight of 4 hogsheads of sugar, weighing as follows, viz.: No. 1, 9 cwt. 2 qrs. 21 lb. No. 2, 10 cwt. 3 qrs. 23 lb. No. 3, 8 cwt. 2 qrs. 25 lb. No. 4, 9 cwt. 3 qrs. 17 lb. Ans. 39 cwt. 1 qr. 2 lb.

51. A section of western land embraces a square mile. How many square rods are there in it? Ans. 102400 rods.

52. A man bought 3 parcels of wood. The first contained 2 cords, 84 feet, 864 inches; the second, 5 cords, 36 ft. 1456 in.; and the third 96 cords, 84 feet. How much did he buy in all? Ans. 104 cords, 77 ft. 592 in.

53. A farmer bought 3 fields which measure as follows, viz.: 17 acres, 2 roods, 18 poles; 39 acres, 1 rood, 16 poles; 12 acres, 36 poles. How much land is there in the three fields? Ans. 69 acres, 30 rds.

54. From a piece of broad cloth, containing 56 yds. 2 nls. a tailor made 3 suits, each of 6 yds. 3 qrs. How much remained in the piece? Ans. 35 yds. 3 qrs. 2 nls.

55. The earth performs one revolution round the sun in 365 days, 5 h. 48 m. 56 sec. How long does it take for it to make 8 revolutions? and how much does that differ from 8 Julian years?

Ans. 2921 d. 22 h. 31 m. 28 sec.; and 1 hour 28 m. 32 sec.

56. Supposing the national debt of England to be 1900 millions of dollars; how long will it take to count it at the rate of 50 dollars a minute, reckoning without intermission 12 hours a day and 365 days to the year?

Ans. 144 yrs. 217 d. 9 h. 20 m.

ALIQUOTS, OR PRACTICE.

INDUCTIVE EXERCISES FOR THE SLATE.

1. If sugar costs a dollar a pound, what would 45 lbs. cost?

2. Then if it costs only $\frac{1}{8}$ of a dollar a lb. what will 45 lbs. cost? Ans. It will cost $\frac{1}{8}$ as much, which is $5\frac{5}{8}$.

3. If apples cost $\frac{1}{4}$ of a dollar a bushel, what will 75 bushels cost? Ans. *If they cost* \$1 *a bushel*, 75 *bushels would cost* \$75; *but if they cost* $\frac{1}{4}$ *of a dollar a bushel*, 75 *bushels will cost* $\frac{1}{4}$ *of* \$75, *which is* $18\frac{3}{4}$.

Perform the following in the same manner.

4. If potatoes cost $\frac{1}{8}$ of a dollar a bushel, what will 27 bushels cost?

5. If shirting cost $\frac{1}{9}$ of a dollar a yard, what will 38 yards cost?

6. What is the cost of 44 yds., at $\frac{1}{10}$ of a dollar a yd.?

7. What is the cost of 73 yds., at $\frac{1}{8}$ of a dollar a yd.?

8. What is the cost of 104 yds., at $\frac{1}{5}$ of a dollar a yd.?

When one number will exactly measure another number, the measuring number is called an aliquot part of the other The following is a

TABLE OF THE ALIQUOT PARTS OF A DOLLAR.

50 cts. = $\frac{1}{2}$ dollar.	$12\frac{1}{2}$ cts. = $\frac{1}{8}$ dollar.
$33\frac{1}{3}$ cts. = $\frac{1}{3}$ dollar.	10 cts. = $\frac{1}{10}$ dollar.
25 cts. = $\frac{1}{4}$ dollar.	$6\frac{1}{4}$ cts. = $\frac{1}{16}$ dollar.
20 cts. = $\frac{1}{5}$ dollar.	5 cts. = $\frac{1}{20}$ dollar.
$16\frac{2}{3}$ cts. = $\frac{1}{6}$ dollar.	4 cts. = $\frac{1}{25}$ dollar.

By the use of this table tell the cost of

9. 16 lbs. of sugar, at $16\frac{2}{3}$ cts. per lb. Ans. \$2.66$\frac{2}{3}$.

10. 24 bush. of apples, at 50 cts. per bush. Ans. $12.00

11. 43 yds. of cloth, at 20 cts. per yd. Ans. $8.60.

12. 137 pts. of nuts, at 5 cts. per pt. Ans. $6.85.

13. 28 brooms, at 33⅓ cts. per broom. Ans. $9.33⅓.

14. 49 yds. of calico, at 10 cts. per yd. Ans. $4.90.

15. 74 oranges, at 6¼ cts. per orange. Ans. $4.62½.

16. What part of a cwt. is 2 qrs.? Then if 1 cwt. of sugar costs 8 dollars, what will 2 qrs. of a cwt. cost?

Ans. ¼ of $8, which is $4.

17. If 2 qrs. cost $4, what will 1 qr. cost?

18. If 1 qr. cost $2, what will 7 lbs. or ¼ of a qr. cost?

Ans. ¼ of $2.00, which is 50 cts.

19. In the last three sums, we have found the cost of 2 qrs. of a cwt. to be $4; the cost of 1 qr. to be $2, and the cost of 7 lbs. to be 50 cts. What then must be the cost of the whole 3 qrs. 7 lbs.?

Ans. $4.00 + $2.00 + 50 cts., or $6.50.

For assisting the pupil in sums of this kind, we present the following

TABLE OF ALIQUOT PARTS OF A CWT.

56 lbs. = ½ cwt.	14 lbs. = ⅛ cwt.
28 lbs. = ¼ cwt.	8 lbs. = $\frac{1}{14}$ cwt.
16 lbs. = ⅛ cwt.	7 lbs. = $\frac{1}{16}$ cwt.

And again.

28 lbs. = ½ of 56 lbs.	8 lbs. = ½ of 16 lbs.
14 lbs. = ¼ of 56 lbs.	4 lbs. = ¼ of 16 lbs.
7 lbs. = ⅛ of 56 lbs.	2 lbs. = ⅛ of 16 lbs.
8 lbs. = $\frac{1}{7}$ of 56 lbs.	7 lbs. = ½ of 14 lbs
14 lbs. = ½ of 28 lbs.	2 lbs. = $\frac{1}{7}$ of 14 lbs.
7 lbs. = ¼ of 28 lbs.	4 lbs. = ½ of 8 lbs.
4 lbs. = $\frac{1}{7}$ of 28 lbs.	2 lbs. = ¼ of 8 lbs.

By the use of this table tell the cost of

20. 56 lbs. of sugar, at $15.00 per cwt. Ans. $7.50.

21. 28 ¼lbs. of hops, at $8.00 per cwt. Ans. $2.00.

22. 16 lbs. of coffee, at $25.00 per cwt. Ans. $3.57 1/7.

23. 14 lbs. of tobacco, at $14.31 per cwt. Ans. $1.78 2/7.

24. What is the cost of 3 cwt. 3 qrs. 17 lbs. of sugar, at $9.00 a cwt.?

2 qrs. will cost ½ of	9.00	= cost of 1 cwt.
	3	
	27.00	= cost of 3 cwt.
1 qr. " " ½ of	4.50	= " " 2 qrs.
14 lbs. " " ½ of	2.25	= " " 1 qr.
2 lbs. " " ⅐ of	1.12	= " " 14 lbs.
1 lb. " " ½ of	1 6	= " " 2 lbs.
	8	= " " 1 lb.
Ans.	$35.11	= cost of 3 C. 3 qrs. 17 lbs.

25. What is the cost of 27 lbs. 10 oz. of iron, at $9.25 per cwt.?

16 lbs. will cost 1/7 of	9.25	= cost of 1 cwt
8 lbs. " " ½ of	1.32	= " " 16 lbs.
2 lbs. " " ¼ of	6 6	= " " 8 lbs.
1 lb. " " ½ of	1 6	= " " 2 lbs.
8 oz. " " ½ of	8	= " " 1 lb.
2 oz. " " ¼ of	4	= " " 8 oz.
	1	= " " 2 oz.
Ans.	$2.27	= cost of 27 lbs. 10 oz.

DEFINITION.

1. We have seen on page 169, that in some cases, when we wish to multiply by a denominate number, we may reduce all the denominations of the multiplier to their lowest denomination. But this can happen only when the price is affirmed of that lowest denomination. Thus, to find the

value of cwt., qrs. and lbs., at so much a *lb.*, we reduce the cwt. and qrs. to lbs.; but if at so much a cwt., it would not answer to reduce to lbs.

2. When there are denominations smaller than the integer of which the price is asserted, the value of the quantity represented by those smaller denominations, is found by *aliquot parts.*

3. An *aliquot part* of any number, is that which forms an exact part of it; that is, a number which is an exact measure of it.

4. The method of calculating by aliquots is called PRACTICE, from its adaptation to frequent use in business.

There are two cases in Practice.

1. *When the price is an aliquot part of the integer.*

RULE.

First, See how many of the denomination, that you buy or sell by, is in the given quantity; and write down the *same number of dollars.*

Second, If the given price is an aliquot part of a dollar, find the same part of the number of dollars just written.

Third, But if the price is not an aliquot part of a dollar, find first the greatest aliquot it contains; and then, with the remainder of the price, find the greatest aliquot that *it* contains of the preceding aliquot; and so on, till all the price has been taken.

APPLICATION.

1. WHAT is the cost of 63 yds. of cloth, at $3.62½ per yd.?

Ans. If the cost was $1 a yard, the whole cost would be $63. If the cost was 3 dollars a yard, the whole cost would be 3 times $63. If the cost was 50 cts. per yard, the whole cost would be ½ of $63. If the cost was 12½ cts., the whole cost would be ¼ of $31.50, &c.

50 cts. = ½)	63.00
	3
	189.00 = at $3.
12½ = ¼)	31.50 = at 50 cts.
	7.87½ at 12½ cts.
	228.37½

What is the answer in each of the following sums:

2. 1250 yards, at 25 cts. per yard?	Ans. $312.50.
3. 640 yds., at 6¼ cts. per yd.?	Ans. $40.00.
4. 270 yds., at 12½ cts. per yd.?	Ans. $33.75.
5. 540 yds., at 10 cts. per yard?	Ans. $54.00.
6. 270 yds., at $2.33⅓ per yard?	Ans. $630.00.
7. 360 yds., at 25 cts. per yard?	Ans. $90.00.
8. 240 yds., at 33⅓ cts. per yard?	Ans. $80.00.
9. 25 yds., at $1.75 per yard?	Ans. $43.75.
10. 420 yds., at $4.83 per yard?	Ans. $2028.60.
11. 221 yds., at $1.10 per yard?	Ans. $243.10.
12. 336 yds., at $2.44 per yard?	Ans. $819.84.
13. 275 yds., at $3.37½ per yard?	Ans. $928.12½.
14. 410 yds., at $5.87½ per yard?	Ans. $2408.75.

2. *When in the quantity there are denominations less than the integer.*

RULE.

Multiply the price by the integers, or whole numbers, in the quantity; and, if there are any lower denominations in the quantity, take the greatest aliquot they contain of the integer, and find its correspondent part of the price. If there is still a part of the quantity left, take its greatest aliquot of the preceding aliquot, and find its correspondent part of the price. And so on, till all the quantity is taken. Then, add the several results together.

APPLICATION.

1. WHAT will 73 yds. 3 qrs. 2 nls. come to, at $5.60 per yard? Ans. $413.70.

2. What will 16 cwt. 2 qrs. 17 lbs. of sugar amount to, $11.75 a cwt.? Ans. $195.65 +

3. What will 11 cwt. 2 qrs. 6 lbs. of tobacco come to, at $4.66 a cwt.? Ans. $53.83 +

4. What will 18 gals. 1 qt. 1½ pts. of wine come to, at $1.38 per gallon? Ans. $25.44 +

5. What is the cost of 37 yds. 1 qr. 3 n. of cloth, at 33 cts. a yard? Ans. $12.35 +

6. What is the value of a load of hay, weighing 1 T. 3 cwt. 1 qr., at $15 a ton? Ans. $17.43 +

7. What is the value of a barrel of rice, weighing 3 cwt. 1 qr. 21 lbs., at $4.10 a cwt.? Ans. $14.09 +

8. What will 2¾ tons of coal cost, at $6 a ton? Ans. $16.50.

9. What is the value of 17 acres 101 rods of land, at $180 an acre? Ans. $3173.62½.

10. What is the cost of $6\frac{9}{30}$ barrels of pork, at $11 a barrel? ☞ $\frac{6}{30} = \frac{1}{5}$, and $\frac{3}{30} = \frac{1}{2}$ of $\frac{6}{30}$. Ans. $69.30.

11. What is the cost of $18\frac{9}{16}$ pounds of butter, at 30 cts. a pound? Ans. $5.56 +

12. What will $9\frac{18}{42}$ bushels of salt come to, at 28 cents a bushel? Ans. $2.639 +

13. What is the cost of 6 cwt. 3 qrs. 9 lbs. of sugar, at 3£. 4 s. 6 d. a cwt.? Ans. 22£. 6¼ d.

14. What is the cost of 17 cwt. 1 qr. 27 lbs. of tobacco, at 1£. 18 s. 10 d. per cwt.? Ans. 33£. 19 s. 2½ d.

15. What is the value of 9 cwt. 1 qr. 13 lbs. of rice, at 1£. 10 s. 4½ d. per cwt.? Ans. 14£. 4 s. 5¾ d.

16. What is the value of 4 cwt. 3 qrs. 6 lbs. of iron, at 2£. 5 s. 3 d. a cwt.? Ans. 10£. 17 s. 4 d.

17. If the use of 100 dollars for 1 year is worth 6 dollars, what is the use of it worth for 1 year 8 months and 10 days? Ans. $10.33¼.

VULGAR FRACTIONS

Reduction to Lower Terms.

Under the rule on page 108, we gave instruction for finding divisors that are not higher than 12. But it very often happens that the common divisor for both terms of the fraction, is more than 12. We therefore give the following

Rule for finding Common Divisors.

Divide the greater number by the less, and then divide the divisor by the remainder; and thus continue dividing the last divisor by the last remainder, till nothing remains. The last divisor will be the greatest common divisor.

To find the common divisor of more than two numbers, find first for two; and then for another and the number just found, and so on through them all.

Examples.

1. What is the greatest common divisor of $\frac{315}{1449}$?

Ans. *As the divisor will divide* 315, *it will also divide* 4 *times* 315, *or* 1260. *But as it will divide* 1449, *it will divide the difference between* 1260 *and* 1449, *which is* 189. *Again as it will divide both* 189 *and* 315, *it will divide their difference, which is* 126, *&c.* The answer is 63.

315)1449(4
1260
189)315(1
189
126)189(1
126
63)126(2
126
0

2. Find the greatest common divisor of $\frac{7631}{264115}$, and reduce it. Ans. $\frac{13}{47}$.

3. Find the divisor of $\frac{42237}{753582}$, and reduce it. Ans. $\frac{19}{34}$.

4. Reduce $\frac{266}{323}$ to its lowest terms. Ans. $\frac{14}{17}$.

5. Reduce $\frac{1257}{2093}$ to its lowest terms. Ans. $\frac{3}{7}$.

6. Reduce $\frac{2181}{15994}$ to its lowest terms. Ans. $\frac{3}{22}$.

7. Reduce $\frac{10386}{16733}$ to its lowest terms. Ans. $\frac{18}{31}$.

REDUCTION OF FRACTIONS TO COMMON DENOMINATORS.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. A BOY bought one book for a half of a dollar, and another book for a quarter of a dollar. How much did he give for both books?

2. A lady bought at one time $\frac{1}{2}$ of a yard of cloth, and at another time $\frac{1}{4}$ of a yard. How many quarters of a yard did she buy at both times?

3. How can you change a half into fourths?

4. A gentleman owned $\frac{1}{3}$ of a ship, and his son owned $\frac{1}{6}$ of her. How many sixths did they both own?

5. How can you change a third into sixths?

6. A boy gave $\frac{1}{5}$ of an apple to his brother, and $\frac{1}{10}$ to his sister. How many tenths did he give away?

7. How do you change a fifth into tenths?

8. A son received from his father $\frac{3}{8}$ of an apple, and from his mother $\frac{1}{4}$ of an apple. How many eighths had he?

9. How do you change a fourth into eighths?

10. How many $\frac{1}{6}$ are in $\frac{1}{3}$? $\frac{2}{3}$? $\frac{5}{6}$?

11. How many $\frac{1}{8}$ are in $\frac{1}{4}$? $\frac{1}{2}$? $\frac{3}{4}$?

12. How much is $\frac{1}{8}$ and $\frac{1}{4}$? $\frac{1}{2}$ and $\frac{3}{4}$?

13. How many $\frac{1}{9}$ are in $\frac{1}{3}$? $\frac{2}{3}$? $\frac{4}{9}$?

14. How much is $\frac{1}{9}$ and $\frac{1}{3}$? $\frac{2}{3}$ and $1\frac{1}{9}$?

15. How many $\frac{1}{15}$ are in $\frac{1}{3}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{2}{3}$? $\frac{3}{5}$?

16. How much is $\frac{1}{15}$ and $\frac{1}{3}$? $\frac{1}{15}$ and $\frac{2}{3}$? $\frac{1}{15}$ and $\frac{3}{5}$?

17. How many $\frac{1}{12}$ are in $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{2}{3}$? $\frac{3}{4}$?

18. How much is $\frac{1}{12}$ and $\frac{1}{2}$? $\frac{1}{12}$ and $\frac{1}{3}$? $\frac{1}{12}$ and $\frac{1}{4}$? $\frac{1}{12}$ and $\frac{1}{6}$? $\frac{1}{12}$ and $\frac{2}{3}$? $\frac{1}{12}$ and $\frac{3}{4}$?

19. What parts are made by cutting a *half* into 2 pieces?

20. What parts are make by cutting a *fourth* into 2 pieces? What by cutting *eighths* into 2 pieces?

21. What parts are made by cutting *thirds* into 2 pieces? What by cutting thirds into 3 pieces?

22. What parts are made by cutting fourths into 2 pieces? Into 3 pieces? Into 4 pieces? Into 5 pieces?

23. What parts are made by cutting fifths into 2 pieces? Into 3 pieces? Into 4 pieces? Into 5 pieces?

24. Now then, to what other denomination can you change $\frac{1}{3}$? Ans. $\frac{1}{3}$ can be changed to *sixths, ninths, twelfths,* &c.

25. To what higher denomination can you change $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

In the first part of this lesson the pupil has found that he cannot add different kinds of things into a sum of one kind. As 3 apples and 4 pears cannot make 7 apples, or 7 pears; so, 2 thirds and 3 fourths cannot make 5 thirds, or 5 fourths. But, we have found that there is a method by which we *may* add 2 thirds and 3 fourths. For, thirds can be changed into *sixths, ninths, twelfths, fifteenths, eighteenths,* &c.; and fourths can be changed into *eighths, twelfths, sixteenths, twentieths,* &c. Therefore, although both *thirds* and *fourths* cannot be changed to *sixths, ninths,* or *eighths;* they *can* both be changed to *twelfths.* This is called reducing fractions to a *common denominator.*

26. To what common denominator can you change $\frac{1}{3}$ and $\frac{1}{5}$? $\frac{1}{4}$ and $\frac{1}{5}$? $\frac{1}{4}$ and $\frac{1}{6}$? $\frac{1}{5}$ and $\frac{1}{6}$? $\frac{1}{6}$ and $\frac{1}{7}$? $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$? $\frac{1}{3}$ and $\frac{1}{8}$? $\frac{1}{4}$ and $\frac{1}{9}$? $\frac{1}{3}$ and $\frac{1}{12}$?

Note.—In the last example, $\frac{1}{3}$ may be changed to $\frac{1}{12}$. Therefore the common denominator is $\frac{1}{12}$.

27. How much is $\frac{1}{5}$ and $\frac{3}{4}$? $\frac{3}{5}$ and $\frac{1}{4}$?

28. Subtract $\frac{1}{4}$ from $\frac{3}{5}$. $\frac{3}{4}$ from $\frac{4}{5}$. $\frac{3}{9}$ from $\frac{5}{6}$.

29. Add $\frac{2}{3}$ and $\frac{2}{9}$. $\frac{3}{6}$ and $\frac{5}{9}$. $\frac{3}{6}$ and $\frac{2}{9}$.

30. Subtract $\frac{3}{7}$ from $\frac{1}{2}$. $\frac{1}{2}$ from $\frac{6}{7}$. $\frac{3}{4}$ from $\frac{5}{6}$.

31. Add $\frac{4}{7}$ and $\frac{3}{4}$. $\frac{3}{8}$ and $\frac{5}{6}$. $\frac{5}{6}$ and $\frac{2}{9}$.

32. Subtract $\frac{3}{5}$ from $\frac{5}{8}$. $\frac{2}{5}$ from $\frac{3}{7}$. $\frac{7}{8}$ from $1\frac{1}{2}$.

INDUCTIVE EXERCISES FOR THE SLATE.

1. A FATHER divided his estate between his three SONS in the following manner: To the eldest he gave $\frac{3}{12}$ of it; to the second, $\frac{4}{9}$ of it; and to the third, the remainder. What part of it did he give the third?

Ans. We have seen (pages 145 and 146,) that we can multiply both terms of a fraction, and thereby obtain another fractional expression that represents the same value. This principle will assist us in finding the answer to this question. Thus, we can represent $\frac{3}{12}$ either as it is, or as $\frac{6}{24}$, or as $\frac{9}{36}$, or as $\frac{12}{48}$, &c. We can also represent $\frac{4}{9}$ either as it is, or as $\frac{8}{18}$, or as $\frac{12}{27}$, or as $\frac{16}{36}$, &c. And by this means, we find that instead of saying, the first had $\frac{3}{12}$ and the second had $\frac{4}{9}$; we may say, the first had $\frac{9}{36}$, and the second had $\frac{16}{36}$. The shares of the first and second sons can now be added together; making $\frac{25}{36}$. Of course, $\frac{25}{36}$ taken from the whole or $\frac{36}{36}$, will leave the share of the third SON. It is $\frac{11}{36}$.

Hence it appears, that the chief part of the operation in this sum and those which are similar to it, is the

REDUCTION OF FRACTIONS TO A COMMON DENOMINATOR.

2. To what common denominator can you change the following fractions—$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{8}$?

Ans. The denominator $\frac{2}{2}$ may be multiplied by 6 to make $\frac{12}{12}$; the denominator $\frac{3}{3}$ may be multiplied by 4 to make $\frac{12}{12}$; the denominator $\frac{4}{4}$ may be multiplied by 3 to make $\frac{12}{12}$; the denominator $\frac{8}{8}$ may be multiplied by 2 to make $\frac{12}{12}$. Hence, all the four fractions may be reduced to $\frac{12}{12}$.

After having determined to make $\frac{}{12}$ the denominator; we know that as we have multiplied each *given* denominator to make it, we must multiply its numerator by the same number. Therefore, *Both terms of* $\frac{1}{2} \times$ *by* $6, = \frac{6}{12}$. *Both terms of* $\frac{2}{3} \times 4 = \frac{8}{12}$. *Both terms of* $\frac{3}{4} \times 3 = \frac{9}{12}$. *Both terms of* $\frac{4}{6} \times 2 = \frac{8}{12}$. *Therefore the fractions become* $\frac{6}{12}, \frac{8}{12}, \frac{9}{12}, \frac{8}{12}$.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{6} = \frac{6}{12}, \frac{8}{12}, \frac{9}{12}, \frac{8}{12}$.

3. Reduce to a common denominator the fractions $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$.

Ans. In this sum, it is plain that both $\frac{}{2}$ and $\frac{}{4}$, can be multiplied to make $\frac{}{8}$. Therefore the fractions will be $\frac{4}{8}, \frac{6}{8}, \frac{5}{8}$.

4. Reduce to a common denominator $\frac{3}{4}, \frac{7}{8}, \frac{5}{16}$.
5. Reduce to a common denominator $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$.
6. Reduce to a common denominator $\frac{2}{3}, \frac{3}{4}, \frac{7}{12}$.
7. Reduce to a common denominator $\frac{2}{3}, \frac{1}{2}, \frac{2}{15}$.
8. Reduce to a common denominator $\frac{1}{8}, \frac{5}{8}, \frac{3}{8}, \frac{7}{18}$.
9. Reduce to a common denominator $\frac{1}{2}, \frac{3}{4}, \frac{2}{5}, \frac{9}{10}$.

Ans. In this sum, all of the first three denominators cannot be multiplied to make the greatest, which is $\frac{}{10}$. But if we double $\frac{}{10}$ and make it $\frac{}{20}$, we shall find that the three other denominators can be changed to the same.

10. Reduce to a common denominator $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$.
11. Reduce to a common denominator $\frac{5}{6}, \frac{4}{15}, \frac{3}{5}$.
12. Reduce to a common denominator $\frac{7}{15}, \frac{1}{6}, \frac{4}{7}$.
13. Reduce to a common denominator $\frac{4}{9}, \frac{5}{18}, \frac{3}{4}$
14. Reduce to a common denominator $\frac{7}{18}, \frac{1}{4}, \frac{5}{9}$.
15. Reduce to a common denominator $\frac{3}{8}, \frac{3}{5}, \frac{3}{16}, \frac{3}{4}$.
16. Reduce to a common denominator $\frac{4}{9}, \frac{5}{12}, \frac{3}{4}$.

Ans. In this sum, the two smallest denominators cannot be multiplied to make $\frac{}{12}$, (which is the highest,) nor its double, which is $\frac{}{24}$; but they can be multiplied to make three times the highest, which is $\frac{}{36}$.

17. Reduce $\frac{3}{16}, \frac{7}{12}, \frac{5}{6}, \frac{1}{8}$, to a common denominator.
18. Reduce $\frac{5}{6}, \frac{3}{5}, \frac{4}{15}, \frac{3}{20}$, to a common denominator.
19. The pupil by this time will see, that in order to find

a common denominator of several fractions, he must first ascertain whether each of the smaller denominators can be multiplied to make the largest. If they cannot be, he tries whether they can be multiplied to make *twice the largest*, *three times the largest*, or *four times the largest*, *&c.*

19. The most common method is to multiply the largest by the *other* denominator; because then it is certain that the other denominator may be multiplied by the *largest* to make the same product. By this principle reduce $\frac{3}{4}$ and $\frac{2}{7}$ to a common denominator.

20. Reduce $\frac{4}{5}$ and $\frac{4}{9}$ to a common denominator.

21. Reduce $\frac{5}{12}$ and $\frac{3}{7}$ to a common denominator.

22. Reduce $\frac{8}{9}$ and $\frac{4}{10}$ to a common denominator.

23. Reduce $\frac{9}{11}$ and $\frac{4}{5}$ to a common denominator.

24. Reduce $\frac{7}{13}$ and $\frac{7}{13}$ to a common denominator.

25. Reduce $\frac{4}{15}$ and $\frac{3}{22}$ to a common denominator.

26. Reduce $\frac{7}{12}$ and $\frac{9}{23}$ to a common denominator.

27. Reduce $\frac{4}{19}$ and $\frac{9}{33}$ to a common denominator.

28. Reduce $\frac{18}{19}$ and $\frac{21}{22}$ to a common denominator.

29. It is plain, that when there are *more than two* denominators, if we multiply *all* the denominators together, the *product* would be a common denominator. Reduce $\frac{5}{6}$, $\frac{2}{5}$, and $\frac{3}{17}$ to a common denominator.

Ans. *Both terms of* $\frac{5}{6} \times 5$ *and then by* $17 = \frac{425}{510}$; *both terms of* $\frac{2}{5} \times 6$ *and then by* $17 = \frac{204}{510}$; *both terms of* $\frac{3}{17} \times 6$ *and then by* $5 = \frac{90}{510}$.

$\frac{5}{6}, \frac{2}{5}, \frac{3}{17} = \frac{425}{510}, \frac{204}{510}, \frac{90}{510}$.

30. Reduce $\frac{2}{13}$, $\frac{3}{7}$, and $\frac{4}{9}$ to a common denominator, and tell their sum. Ans. $\frac{841}{819}$.

31. Reduce $\frac{7}{9}$, $\frac{5}{11}$, and $\frac{2}{3}$ to a common denominator, and tell their sum. Ans. $1\frac{427}{693}$.

32. Reduce $\frac{7}{9}$, $\frac{4}{13}$, and $\frac{8}{7}$ to a common denominator, and tell their sum. Ans. $1\frac{404}{819}$.

33. Reduce $\frac{4}{9}$, $\frac{5}{14}$, and $\frac{2}{5}$ to a common denominator, and tell their sum. Ans. $1\frac{253}{630}$.

DEFINITIONS.

1. NUMBERS, when compared with one another, are either *prime* or *composite*.

2. A PRIME NUMBER is one that cannot be made by multiplying any two other numbers together; such as 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, &c.

3. A COMPOSITE NUMBER is one that may be obtained by multiplying two or more numbers together; such as $4 = 2 \times 2$; $6 = 2 \times 3$; $8 = 2 \times 4$; $9 = 3 \times 3$; $16 = 4 \times 4$; $48 = 6 \times 8$; &c.

4. Those numbers that are multiplied together to make a composite number, are called FACTORS of that number. Thus, 24 is composed of the factors 2 and 12, or of the factors 3 and 8, or of the factors 4 and 6, or of the factors 2, 2, and 6, or of the factors 2, 3, and 4, or of the factors 2, 2, 2, and 3.

5. When we compare a composite number with *one* of its factors, we call the composite number a MULTIPLE of the factor. Thus, 6 is a multiple of 2 or of 3; 12 is a multiple of 2, or 3, or 4, or 6; &c.

6. When two or more numbers will measure a larger number, that larger number is called the COMMON MULTIPLE of those smaller ones. Thus, 36 is a common multiple of 2, 4, 6, 9, 12, 18.

7. When, with a composite number, we compare one of its factors, we call the factor a MEASURE of the larger number. Thus, 2 is a measure of 4, or 6, or 8, &c.; 3 is a measure of 6 or 9; 4 is a measure of 8, or 12, or 16; &c.

What is a prime number? What are the first ten of them? Ascertain which are the next ten. What is a composite number? Mention some composite numbers besides the examples. What are factors? Mention the factors of 6, 18, 36, 48, and some others. What is a multiple? Mention a few multiples of 7, 3, 5, 9, 12. What is a common multiple? What is a measuring number?

8. When a number is a factor of 2 or more numbers, it is called a COMMON MEASURE of those numbers. Thus, 6 is a common measure of 12, and 18, and 24; 9 is a common measure of 36, and 63, and 81.

9. When two numbers have not a common measure, they are said to be *prime to one another*. Thus, 4 and 9 are prime to one another; but 6 and 9 are not prime to one another.

10. A common multiple of the denominators of several fractions, is sometimes called the COMMON DENOMINATOR.

RULE

For finding the Common Multiple, and reducing Fractions to a Common Denominator.

LET us suppose that each of the fractions is reduced to its lowest terms. Then,

First, See if the greatest denominator mentioned, is a multiple of each of the others. If it is, it will be the common multiple and denominator.

Second, If the greatest denominator mentioned, is not a multiple of each of the others; consider how many times it must be multiplied to become so.

1. *If all the denominators are prime to each other,*

One denominator must be multiplied by *all the* others, to make a common multiple.

Then, Each numerator must be multiplied by all the denominators except its own, for its new numerator.

Note.—For common purposes this method is sufficiently convenient whether the denominators are prime or not.

When is a number called a common measure? When are numbers said to be prime to one another? What is a common denominator? What preliminary step is proper before trying for the common denominator? What is the first step in finding the common denominator? What the second? How can this be done? What is the next step?

2. RULE FOR FINDING THE LEAST COMMON MULTIPLE.

Having two numbers, divide one of them by the greatest common divisor. Then multiply the other by the *quotient*, instead of the number itself.

If there are more than two numbers, find the least common multiple of two; and then of the result and another number, and so on to the last.

SUMS FOR EXERCISE.

1. What is the least common multiple of 72, 96, 33?

Ans. 72 *and* 96 *can both be divided by* 12. *The quotients are* 6 *and* 8. *These can both be divided by* 2. *The quotients are* 3 *and* 4. 3 *times* 96 = 288.

288 *and* 33 *can both be divided by* 3. *The quotients are* 96 *and* 11. 288 × 11 = 3168.

72 96 } 288
6 8 }
3 4 }

288 33 } 3168
96 11 }

2. Find the least common multiple of 16 and 24. Ans. 48.
3. Find the least common multiple of 21 and 49. Ans. 147.
4. Find the least common multiple of 30 and 35. Ans. 210.
5. Find the least common multiple of 27 and 51. Ans. 459.
6. Find the least common multiple of 28 and 98. Ans. 196.
7. Find the least common multiple of 3, 8, 10. Ans. 120.
8. Find the least common multiple of 12, 9, 8. Ans. 72.
9. Find the least common multiple of 14, 21, 28. Ans. 84.

Note.—When either of the numbers is a measure of another of the numbers, the number that is a measure may be omitted in the calculation; as follows:

10. What is the least common multiple of 9, 12, 15, and 27? Ans. 540.
11. What is the least common multiple of 6, 15, 18, and 24? Ans. 360.
12. What is the least common multiple of 9, 16, 18, and 24? Ans. 144.
13. What is the least common multiple of 5, 8, 10, and 18? Ans. 360.

To reduce fractions to the least common denominator,

We find the least common multiple of the denominators. Then, if there are but two fractions, the first numerator must be multiplied by the last quotient, and the second numerator by the first quotient.

But if there are more than two fractions, divide the common denominator by each given denominator, and multiply its numerator by the quotient. Or, as in the second example.

SUMS FOR EXERCISE.

1. Reduce $\frac{8}{5}$ and $\frac{13}{15}$ to their least common denominator.

Ans. *Both* 9 *and* 15 *can be divided by* 3; *and make the quotients* 3 *and* 5. *Both terms of* $\frac{8}{5} \times 5 = \frac{40}{45}$. *Both terms of* $\frac{13}{15} \times 3 = \frac{39}{45}$. *The answers are* $\frac{40}{45}$ *and* $\frac{39}{45}$.

$\frac{8}{5}, \frac{13}{15} = \frac{40}{45}, \frac{39}{45}$.

2. Reduce $\frac{5}{6}$, $\frac{2}{9}$, $\frac{5}{8}$ to their least common denominator.

Ans. 6 *and* 9 *make the quotients* 2 *and* 3; *and their common multiple is* 18. 18 *and* 8 *make the quotients* 9 *and* 4; *and their common multiple is* 72.

Both terms of $\frac{5}{6} \times 3$ *and* $4 = \frac{60}{72}$. *Both terms of* $\frac{2}{9} \times 2$ *and* $4 = \frac{16}{72}$. *Both terms of* $\frac{5}{8} \times 9 = \frac{45}{72}$. *Therefore the answer is*

$\frac{5}{6}, \frac{2}{9}, \frac{5}{8} = \frac{60}{72}, \frac{16}{72}, \frac{45}{72}$

2 3 18 9 8 4

3. Reduce $\frac{11}{24}$, $\frac{29}{32}$, and $\frac{31}{36}$ to their least common denominator. ☞ See note p. 174. Ans. $\frac{308}{672}, \frac{609}{672}, \frac{372}{672}$.

4. Reduce $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{12}$, and $\frac{5}{20}$ to their least common denominator. Ans. $\frac{80}{120}, \frac{45}{120}, \frac{50}{120}, \frac{30}{120}$.

5. Reduce $\frac{8}{9}$, $\frac{7}{15}$, and $\frac{13}{20}$. Ans. $\frac{180}{180}, \frac{84}{180}, \frac{117}{180}$.

6. Reduce $\frac{5}{9}$, $\frac{6}{15}$, and $\frac{3}{16}$. Ans. $\frac{400}{720}, \frac{288}{720}, \frac{135}{720}$.

7. Reduce $\frac{6}{8}$, $\frac{15}{36}$, $\frac{7}{9}$, and $\frac{5}{16}$. Ans. $\frac{108}{144}, \frac{60}{144}, \frac{112}{144}, \frac{45}{144}$.

8. Reduce $\frac{7}{16}$, $\frac{8}{32}$, and $\frac{11}{44}$. Ans. $\frac{70}{160}, \frac{45}{160}, \frac{44}{160}$.

9. Reduce $\frac{5}{48}$, $\frac{17}{36}$, and $\frac{9}{64}$. Ans. $\frac{140}{1344}, \frac{408}{1344}, \frac{189}{1344}$.

ADDITION AND SUBTRACTION OF VULGAR FRACTIONS.

Rule.

Reduce the fractions to their lowest terms, and bring them to a common denominator; and then add or subtract their numerators as before.

APPLICATION.

1. A lady bought at one time $2\frac{1}{2}$ yds. of cloth, and at another, $3\frac{3}{4}$ yds. How many yds. did she buy in all?
Ans. $6\frac{1}{4}$ yds.

2. A man bought $\frac{3}{4}$ of a dozen of eggs at one time, and $\frac{5}{8}$ of a dozen at another time. How many dozen did he buy? Ans. $1\frac{5}{12}$ doz.

3. A gentleman divided a dollar among his 3 sons, by giving $\frac{3}{10}$ of it to the first, $\frac{3}{10}$ of it to the second, and the rest of it to the third. What part of a dollar did he give the third son? Ans. $\frac{3}{10}$ of a dollar.

4. A man bought coffee at four times, as follows: $11\frac{3}{4}$ lbs.; $7\frac{1}{4}$ lbs.; $9\frac{3}{4}$ lbs.; $12\frac{2}{5}$ lbs. How many pounds in all? Ans. $41\frac{43}{20}$ lbs.

5. A merchant bought a piece of cloth containing 43 yds., and sold at one time $4\frac{3}{4}$ yds., and at another time $7\frac{4}{12}$ yds. How many yards had he left? Ans. $30\frac{11}{12}$ yds.

6. Bought $14\frac{4}{5}$ of a hundred of sugar, and sold $9\frac{5}{8}$ of it. How much remains? Ans. $5\frac{7}{40}$ C.

7. A man gave $\frac{3}{8}$ of his property to one son, $\frac{2}{5}$ of it to another, and the rest to his daughter. What was the daughter's share? Ans. $\frac{9}{40}$.

8. A clergyman was asked how old he was, and replied that he was $25\frac{3}{12}$ years old when he began to preach, and that he had preached $19\frac{9}{12}$ years. How old was he at the time of the inquiry? Ans. $44\frac{21}{12}$ years.

9. A merchant purchased a quantity of sugar, and sold $\frac{3}{11}$ of it to one man, and $\frac{1}{4\frac{1}{5}}$ of it to another. What part of it was sold? Ans. $\frac{158}{330}$.

10. A man sells $\frac{3}{1\frac{1}{5}}$ of a lot of land to one man, and $\frac{2}{5}$ of it to another. What part does he sell? Ans. $\frac{1}{4\frac{1}{5}}$.

11. There is a common pasture, of which one man owns $\frac{3}{20}$; a second, $\frac{10}{38}$; and a third, $\frac{7}{40}$. What part of the pasture is owned by the three? Ans. $\frac{23}{40}$.

12. If one man owns $\frac{2}{3\frac{1}{3}}$ of a ship, and another $\frac{3}{3\frac{1}{3}}$ of it, what is the difference of their shares? Ans. $\frac{23}{1254}$.

13. There is a sum of money, of which A owns $\frac{1}{2}$, B owns $\frac{2}{5}$, and C owns the rest. What part of the sum does C own? Ans. $\frac{1}{10}$.

14. There is a pole standing, so that $\frac{1}{6}$ of it is in the mud, $\frac{4}{7}$ of it in the water, and the rest out of the water. What part of it is out of the water? Ans. $\frac{11}{42}$.

15. A man who was to do a piece of work in 3 days did $\frac{2}{3}$ of it the first day, and $\frac{1}{3}$ of it the second. How much was to be done on the third day? Ans. $\frac{4}{15}$.

16. Suppose I have $\frac{3}{8}$ of a ship that is worth £1500, and that I buy another person's share of her, which is $\frac{4}{10}$. What part of her will belong to me then; and what is it worth? Ans. £1031, 5s.

17. Suppose that I have $\frac{5}{8}$ of a ship which is worth £900, and that I sell $\frac{5}{12}$ of her. What part of her have I left; and what is it worth? Ans. £187, 10s.

18. What is the sum of $\frac{3}{7}+\frac{4}{2}+\frac{3}{3}$? Ans. $3\frac{1}{3}$.

19. What is the difference between $3\frac{7}{9}$ and $2\frac{10}{11}$? Ans. $\frac{88}{99}$.

20. $\frac{5}{6} + \frac{3}{5} + 2\frac{1}{3} = 3\frac{23}{30}$.

21. $\frac{7}{13} - \frac{5}{13} = \frac{19}{130}$.

22. A man has $\frac{3}{5}$ of a ship, and his son has $\frac{5}{18}$ of her. How much do they both own? Ans. $\frac{11}{18}$ of her.

23. A person who owns $\frac{5}{8}$ of a house sells $\frac{1}{4}$ of it. How much has he left? Ans. $\frac{7}{24}$ of it.

24. Reduce 2, 5, $\frac{3}{4}$, $\frac{2}{7}$ to a common denominator, and tell their sum. Ans. $\frac{56}{28}$, $\frac{140}{28}$, $\frac{21}{28}$, $\frac{8}{28}$.

25. Reduce $\frac{3}{4}$, $\frac{5}{6}$, 6, $\frac{8}{5}$ to a common denominator, and tell their sum. Ans. $\frac{27}{30}$, $\frac{25}{30}$, $\frac{216}{30}$, $\frac{24}{30}$.

26. What improper fraction will express the amount of $\frac{8}{14}$, $\frac{3}{11}$, 6, $\frac{5}{9}$, and $\frac{4}{9}$? Ans. $\frac{2849}{693}$.

27. What improper fraction will express the amount of $\frac{9}{14}$, $\frac{3}{18}$, 2, $\frac{7}{8}$? Ans. $\frac{2645}{756}$.

28. What improper fraction will express the amount of $\frac{4}{3}$, $\frac{3}{16}$, 5? Ans. $\frac{1471}{276}$.

29. What improper fraction will express the amount of $\frac{14}{26}$, $\frac{20}{39}$, 7? Ans. $\frac{3761}{481}$.

30. What is the sum of $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$? Ans. $3\frac{11}{20}$.

31. From $\frac{5}{6} + \frac{2}{3}$, take $\frac{1}{2} + \frac{3}{4}$. Ans. $\frac{17}{24}$.

32. Find the sum of $\frac{4}{9} + \frac{3}{5} + \frac{7}{12} + \frac{3}{7}$? Ans. $1\frac{899}{1260}$.

33. Add $\frac{96}{120}$, $\frac{60}{108}$, $\frac{72}{84}$ and $\frac{36}{48}$. Ans. $2\frac{812}{1260}$.

34. From $\frac{51}{52}$ take $\frac{37}{50}$. Ans. $\frac{5}{130}$.

35 From $2\frac{61}{63}$ take $1\frac{13}{17}$. Ans. $1\frac{93}{160}$.

36 $\frac{21}{63} + \frac{24}{192} + \frac{49}{56} + \frac{33}{55} + \frac{30}{32} = 2\frac{41}{60}$.

37. $3\frac{1}{8} + 4\frac{5}{12} + 8\frac{7}{18} + 14\frac{7}{16} = 30\frac{53}{144}$.

38. $\frac{12}{27} + \frac{30}{48} + \frac{77}{84} + \frac{49}{132} = 2\frac{61}{216}$.

MULTIPLICATION OF FRACTIONS;

OR,

FRACTIONS OF FRACTIONS.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. WHAT is $\frac{1}{2}$ of 2 apples? $\frac{1}{2}$ of 2 pieces of an apple? $\frac{1}{2}$ of $\frac{1}{2}$ of an apple?

2. What is $\frac{1}{5}$ of 25? $\frac{1}{5}$ of $\frac{25}{67}$?

3. What is $\frac{1}{7}$ of $\frac{24}{41}$? $\frac{2}{7}$ of $\frac{24}{41}$? $1\frac{2}{7}$ times $\frac{24}{41}$?

4. What is $\frac{1}{3}$ of $\frac{9}{20}$? $\frac{2}{3}$ of $\frac{9}{20}$? $2\frac{2}{3}$ times $\frac{9}{20}$?

5. What is $\frac{1}{8}$ of $\frac{42}{21}$? $\frac{3}{8}$ of $\frac{42}{21}$? $1\frac{1}{8}$ times $\frac{42}{21}$?

6. What is $\frac{1}{4}$ of $\frac{36}{45}$? $\frac{3}{4}$ of $\frac{36}{45}$? $2\frac{1}{4}$ times $\frac{36}{45}$?

7. What is $\frac{1}{5}$ of $\frac{25}{45}$? $\frac{3}{5}$ of $\frac{35}{46}$? $1\frac{1}{5}$ times $\frac{25}{46}$?

8. What is $\frac{1}{6}$ of $\frac{36}{18}$? $\frac{4}{6}$ of $\frac{36}{18}$? $\frac{5}{6}$ of $\frac{36}{18}$?

9. What is $\frac{1}{5}$ of $\frac{63}{30}$? $\frac{4}{5}$ of $\frac{63}{30}$? $\frac{3}{5}$ of $\frac{63}{30}$?

10. How can you divide $\frac{1}{2}$ an apple into 2 parts? Ans. *Cut the half* into 2 pieces.

11. What would those pieces be called? Ans. *Quarters*, or *fourths*.

12. Then what is the $\frac{1}{2}$ of a $\frac{1}{2}$?

13. If an apple were cut into thirds, and each of those thirds into two pieces, what would those pieces be called?

14. If each of those thirds were cut into three pieces, what would those pieces be called? Then what is $\frac{1}{3}$ of $\frac{1}{3}$?

15. If each of those thirds were cut into 4 pieces, what would those pieces be called? Then what is $\frac{1}{4}$ of $\frac{1}{3}$?

16. If each of those thirds were cut into 5 pieces, what would those pieces be called?

17. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{3}$?

18. If each of $\frac{1}{2}$ were cut into 2 equal parts, what would those pieces be called? What is $\frac{1}{2}$ of $\frac{1}{2}$?

19. What if they were cut into 3 equal parts? What, if into 4? 5? 6? 8?

20. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{6}$ of $\frac{1}{4}$? $\frac{1}{8}$ of $\frac{1}{4}$?

21. What parts are made by cutting fifths into 2 pieces? Into 3? Into 5? Into 7? Into 10?

22. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{7}$ of $\frac{1}{3}$? $\frac{1}{10}$ of $\frac{1}{3}$?

23. What is $\frac{1}{2}$ of $\frac{1}{7}$? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{7}$ of $\frac{1}{8}$? $\frac{1}{4}$ of $\frac{1}{9}$? $\frac{1}{8}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{8}$? $\frac{1}{4}$ of $\frac{1}{10}$? $\frac{1}{8}$ of $\frac{1}{7}$? $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{8}$ of $\frac{1}{2}$? $\frac{1}{9}$ of $\frac{1}{8}$? $\frac{1}{8}$ of $\frac{1}{9}$?

24. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{2}{4}$? Ans. 2 times $\frac{1}{12}$ &c.

25. What is $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{2}{4}$ of $\frac{1}{3}$? $\frac{3}{5}$ of $\frac{1}{3}$?

26. What is $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{2}{3}$? $\frac{1}{3}$ of $\frac{3}{3}$?

27. What is $\frac{1}{6}$ of $\frac{1}{7}$? $\frac{3}{6}$ of $\frac{1}{7}$? $\frac{4}{6}$ of $\frac{1}{7}$?

28. What is $\frac{1}{7}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{2}{3}$? $\frac{2}{7}$ of $\frac{2}{3}$?

29. What is $\frac{2}{3}$ of $\frac{1}{6}$? $\frac{2}{3}$ of $\frac{1}{6}$? $\frac{2}{3}$ of $1\frac{2}{8}$?

30. What is $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{3}{4}$ of $\frac{1}{8}$? $1\frac{3}{4}$ times $\frac{1}{8}$?

31. What is $\frac{1}{5}$ of $\frac{1}{6}$? $\frac{4}{5}$ of $\frac{1}{6}$? $\frac{4}{5}$ of $\frac{5}{6}$?

32. What is $\frac{1}{7}$ of $\frac{1}{9}$? $\frac{1}{7}$ of $\frac{4}{9}$? $1\frac{3}{7}$ times $\frac{4}{9}$?

33. What is $\frac{1}{8}$ of $\frac{1}{5}$? $\frac{1}{8}$ of $\frac{4}{5}$? $1\frac{1}{8}$ times $\frac{4}{5}$?

INDUCTIVE EXERCISES FOR THE SLATE.

1. If 3 men give for their board $\frac{12}{8}$ of a dollar a day, what does 1 of them give? And what would 7 men give at that rate? Ans. *One man would give* $\frac{1}{3}$ *of* $\frac{12}{8}$ *of a dollar, which is* $\frac{4}{8}$ *of a dollar; and 7 men would give seven times* $\frac{4}{8}$ *of a dollar, which is* $\frac{28}{8}$ *of a dollar, or* $\$3\frac{4}{8} = \$3\frac{1}{2}$.

2. If 4 sheep are worth $\$32\frac{4}{11}$, what is the worth of 7 sheep at that rate? [See page 91.] Ans. $\$56\frac{7}{11}$.

3. If 7 barrels of flour cost $\$52\frac{1}{2}$, what will 11 barrels cost at that rate? Ans. $\$83\frac{1}{2}$.

4. If 3 apples cost $\frac{5}{12}$ of a shilling, what is the cost of 1 apple?

As we cannot divide the numerator by 3, without a remainder, we multiply both terms, so as to make the numerator a multiple of 3. If we multiply both terms by the number which is to be the divisor, it is certain that the *new* numerator can be divided back by that number.

Ans. *Both terms of* $\frac{5}{12} \times 3 = \frac{15}{36}$; *and* $\frac{1}{3}$ *of* $\frac{15}{36} = \frac{5}{36}$.

$\frac{5}{12} = \frac{15}{36}$ and $\frac{1}{3}$ of $\frac{15}{36} = \frac{5}{36}$

That this method of performing the sum is correct, it will appear by giving the answer in another form. If the thing be divided into 12 equal parts, and then each of those parts into *three*, there would be 36 parts in all. Therefore, $\frac{1}{3}$ *of* $\frac{1}{12}$ *is* $\frac{1}{36}$, *and* $\frac{1}{3}$ *of* $\frac{5}{12}$ *is 5 times* $\frac{1}{36}$ *which* $= \frac{5}{36}$.

5. What is $\frac{1}{7}$ of $\frac{3}{19}$? Ans. $\frac{3}{133}$.
6. What is $\frac{1}{9}$ of $\frac{5}{13}$? Ans. $\frac{5}{117}$.
7. What is $\frac{1}{12}$ of $\frac{7}{18}$? Ans. $\frac{7}{216}$.
8. What is $\frac{1}{14}$ of $\frac{3}{17}$? Ans. $\frac{3}{238}$.
9. What is $\frac{1}{20}$ of $\frac{19}{41}$? Ans. $\frac{19}{820}$.
10. What is $\frac{1}{17}$ of $\frac{8}{25}$? Ans. $\frac{8}{425}$.
11. What is $\frac{1}{8}$ of $\frac{7}{23}$? Ans. $\frac{7}{184}$.
12. What is $\frac{1}{13}$ of $\frac{11}{21}$? Ans. $\frac{11}{273}$.

As in this method of operating, we multiply the numerator by a number, and then immediately divide the product by the *same* number; we may omit that part of the proceeding as useless. Hence, we come to the conclusion, that to multiply one fraction by another, we have only to multiply the two denominators together, and it is done. Perform the following examples in this manner.

13. What is $\frac{1}{4}$ of $\frac{3}{11}$? Ans. $\frac{1}{4}$ *of* $\frac{3}{11} = \frac{3}{44}$. | $\frac{1}{4} \times \frac{3}{11} = \frac{3}{44}$.
14. What is $\frac{1}{9}$ of $\frac{10}{17}$? Ans. $\frac{10}{153}$.
15. What is $\frac{1}{10}$ of $\frac{8}{21}$? Ans. $\frac{8}{210}$.
16. What is $\frac{1}{21}$ of $\frac{3}{28}$? Ans. $\frac{3}{588}$.

17. What is $\frac{6}{10}$ of $\frac{19}{11}$?

Ans. $\frac{1}{10}$ *of* $\frac{19}{11}$ *is* $\frac{19}{110}$, *and* $\frac{6}{10}$ *of* $\frac{19}{11}$ *is* 6 *times* $\frac{19}{110}$, *which is* $\frac{114}{110}$; *which* $= \frac{57}{110}$.

$$\frac{6}{10} \times \frac{19}{11} = \frac{114}{110} = \frac{57}{110}$$

From this sum it appears, that besides multiplying the denominators together for a new denominator, we also multiply the numerators together for a new numerator. Perform the following in this manner.

18. What is $\frac{4}{5}$ of $\frac{7}{9}$? Ans. $\frac{28}{45}$.

19. What is $\frac{9}{3}$ of $\frac{4}{11}$? Ans. $\frac{9}{33}$.

20. What is $\frac{14}{37}$ of $\frac{21}{44}$? Ans. $\frac{637}{1628}$.

21. What is $\frac{8}{25}$ of $\frac{9}{45}$? Ans. $\frac{72}{1125}$.

DEFINITIONS.

1. To multiply one number by another, is to find the number that is formed from the multiplicand, in the same manner that the multiplier is formed from units.

2. Thus, to multiply by 2, is to make the multiplicand, 2 times as large; to multiply by $\frac{1}{2}$, is to make the multiplicand one half as large.

3. Hence, to *multiply by a fraction*, is to take *out of* the multiplicand such *a part* of it as is denoted by the multiplying fraction.

4. When both the multiplier and the multiplicand are fractions, the operation is called *multiplying a fraction by a fraction.*

RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Note.—If the numerator of the multiplicand can be divided by the denominator of the multiplier, it is preferable to do so, and then to multiply that quotient by the

numerator of the multiplier. The number thus obtained, will become the new numerator, which must be put over the denominator of the multiplicand for the answer.

When both the multiplier and the multiplicand are mixed numbers, they must first be changed to improper fractions. But if only one of them is a mixed number, and the integers consist of *several figures*, it is generally best to let it remain so, and do it by page 89 or 97.

APPLICATION OF THE RULE.

1. Multiply $\frac{2}{7}$ by $\frac{5}{8}$. [as in last page.] Ans. $\frac{10}{56} = \frac{5}{28}$.
2. Multiply $\frac{4}{15}$ by $\frac{5}{24}$. Ans. $\frac{1}{18}$.
3. Multiply $3\frac{2}{7}$ by $4\frac{14}{33}$. Ans. $14\frac{124}{231}$.
4. Multiply $472\frac{4}{9}$ by $\frac{4}{11}$. Ans. $171\frac{70}{99}$.
5. Multiply $87\frac{14}{30}$ by $4\frac{11}{15}$. Ans. $371\frac{43}{225}$.
6. Multiply $\frac{4}{4}\frac{27}{93}$ by $\frac{7}{51}$. Ans. $\frac{2989}{30073}$.
7. Multiply $1\frac{347}{868}$ by $94\frac{1}{15}$. Ans. $130\frac{1005}{2604}$.
8. How much is $\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{3}{5}$, added to $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{1}{3}$? Ans. $\frac{24}{120}$.
9. How much is $\frac{5}{7}$ of $\frac{3}{20} - \frac{0}{14}$ of $\frac{3}{22}$? Ans. $\frac{3}{154}$.
10. How much is $\frac{5}{6}$ multiplied by $\frac{3}{7}$ of $\frac{6}{7}$? Ans. $\frac{10}{21}$.
11. What is the product of $\frac{7}{9}$, $\frac{3}{5}$, and $4\frac{5}{14}$? Ans. $2\frac{1}{30}$.
12. How much is $\frac{3}{7}$ of $\frac{4}{9}$ of $\frac{14}{15}$? Ans. $\frac{8}{45}$.
13. If 7 lbs. of sugar cost $\frac{4}{9}$ of a dollar, what is the cost of 1 lb.? And of 3 lbs. at the same rate?
Ans. $\frac{4}{63}$,—and $\frac{4}{21}$ of a dollar.
14. A man traveled $464\frac{4}{5}$ miles in 8 days. How far did he travel in 1 day? And how far would he travel in 15 days at the same rate?
Ans. In one day, $58\frac{1}{10}$ miles; and in 15 days, $871\frac{1}{2}$ miles.
15. If 9 lbs. of rice cost $\frac{3}{7}$ of a dollar, what is the cost of 1 lb.?—and of 15 lbs.? Ans. $\frac{1}{21}$,—and $\frac{5}{7}$ of a dollar.

16. If 8 lbs. of coffee cost $\frac{9}{20}$ of a dollar, what is the cost of 1 lb.?—and of 17 lbs.?

Ans. $\frac{9}{160}$,—and $2\frac{3}{160}$ of a dollar.

17. If 3 lbs. of tea cost $4\frac{7}{16}$ of a dollar, what is the cost of 1 lb.?—and of 19 lbs.? Ans. $\$1\frac{23}{48}$,—and $\$28\frac{5}{48}$.

18. If 15 barrels of flour cost $\$110\frac{7}{8}$, what would 6 barrels cost? [6 barrels will cost $\frac{6}{15}$ of it.] Ans. $\$44\frac{7}{20}$.

19. If 1 lb. of sugar cost $\frac{2}{17}$ of a dollar, what will $11\frac{1}{7}$ lbs. cost? Ans. $\$1\frac{43}{119}$.

20. If 1 lb. of tea costs $\frac{14}{17}$ of a dollar, what is the cost of $9\frac{1}{9}$ lbs.? Ans. $\$7\frac{119}{153}$.

21. If 5 barrels of flour cost $\$25\frac{1}{7}$, what would 13 barrels cost? Ans. $\$66\frac{17}{35}$.

22. If 1 cwt. 2 qrs. 8 lbs. of sugar cost $\$12\frac{4}{7}$, what is the cost of 3 cwt. 1 qr. 16 lbs.? [See p. 133.] Ans. $\$27\frac{1}{7}$.

23. If 7 yds. 1 qr. 2 n. of calico cost $\$1\frac{3}{10}$, what is the cost of 3 qrs. 1 n.? Ans. $\$\frac{169}{1180}$, or $.14\frac{19}{59}$.

Canceling.

In multiplication of fractions it is very often convenient to employ the method of canceling.

Rule.—If there are two equal numbers, one in the numerator and one in the denominator, cross them out, and omit them in the operation. Also, if the same factor is found in *both* a numerator and a denominator, divide them by that factor, and substitute the quotients for the original number.

24. What is $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{8}$.

Ans. *Cancel the* 5*s*; *divide* 4 *and* 8 *by* 4; *and divide* 3 *and* 9 *by* 3. *Then multiplying the numbers that are not crossed, we have* $\frac{1}{6}$ *for the answer.*

$$\frac{\cancel{4}^{1}\times\cancel{5}\times\cancel{3}^{1}}{\cancel{5}\times\cancel{9}^{3}\times\cancel{8}^{2}}=\frac{1}{6}$$

25. How much is $\frac{2}{9}$ of $\frac{3}{5}$ of $\frac{5}{6}$ of $2\frac{23}{7}$? Ans. $\frac{23}{63}$.

26. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$? Ans. $\frac{1}{6}$.

27. What is $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{5}{4}$? Ans. $\frac{3}{8}$.

28. What is the product of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{3}{5}$? Ans. $\frac{6}{126}$.

29. What is the product of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{11}$? Ans. $\frac{4}{11}$.

30. What is the product of $\frac{36}{37}$ of $\frac{21}{48}$ of $\frac{74}{43}$? Ans. $\frac{21}{86}$.

31. How much is $\frac{6}{13}$ of $\frac{13}{36}$ of $\frac{5}{6}$? Ans. $\frac{5}{36}$.

32. How much is $\frac{1}{5}$ of $\frac{5}{8}$ of $\frac{7}{8}$ of $\frac{8}{11}$ of $5\frac{1}{2}$? Ans. $\frac{7}{12}$.

If either of the multipliers is a whole number, it may be called so many *ones*. Thus, $6 = \frac{6}{1}$.

33. How much is 8 times $\frac{3}{5}$ of $\frac{7}{8}$ of 5? Ans. 21.

34. Multiply 9 by $\frac{3}{4}$ of $\frac{6}{5}$. Ans. $8\frac{1}{10}$.

35. Multiply $\frac{2}{5}$ of 5 by $\frac{2}{7}$ of $\frac{3}{8}$ of $4\frac{1}{3}$. Ans. $2\frac{8}{11}$.

36. What is the product of $\frac{3}{7}$, $\frac{4}{9}$, 6, and $\frac{14}{13}$? Ans. $1\frac{1}{13}$.

37. Multiply 9, $\frac{3}{7}$, $\frac{5}{6}$, 2. Ans. $3\frac{8}{14}$.

Reduction of Vulgar Fractions to Denominate Fractions.

1. What is $\frac{1}{8}$ of 3 shillings? Ans. $\frac{3}{8}$s.

2. Reduce and divide 3 shillings into 6 equal parts.
Ans. 3s. = 36d. and $\frac{1}{6}$ of 36d. = 6d.

3. Express, in a fractional form, the division of 6 lbs. of sugar into 8 equal parts. Ans. $\frac{6}{8}$ lb.

4. Divide 6 lbs. into 8 equal parts, and tell how many ounces would be in each part. Ans. 12 oz.

5. Express, in a fractional form, the division of 3 days into 10 equal parts. Ans. $\frac{3}{10}$ day.

6. If 3 days be divided into 10 equal parts, how much time would be in each part? Ans. 7 h. 12 m.

7. Express, in a fractional form, the division of 3£. into 8 equal parts. Ans. $\frac{3}{8}$£.

8. If 3£. be divided into 8 equal parts, how much money would be in each part? Ans. 7s. 6d.

From what was shown on pages 80 and 81, it is evident that the fraction $\frac{3}{8}$ of a pound, may be obtained either by dividing 3£. into 8 equal parts, or by dividing 1 into

8 equal parts, and taking 3 of those parts; that is, $\frac{3}{8}$ of 1£. is equal to $\frac{1}{8}$ of 3£.; because, to express the division of 3£. by 8, we write $\frac{3}{8}$£. But we have just found that $\frac{1}{8}$ of 3£. = 7s. 6d. Therefore, $\frac{3}{8}$ of 1£. must also equal 7s. 6d. In the same manner, $\frac{7}{9}$ of a cwt. must equal $\frac{1}{9}$ of 7 cwt.

9. How much is in $\frac{7}{9}$ of a cwt.?
Ans. $\frac{7}{9}$ cwt. = $\frac{1}{9}$ of 7 cwt. which = 3 qrs. 3 lb. $1\frac{7}{9}$ oz.

10. How much is in $\frac{6}{7}$ of a yard? Ans. 3 qrs. $1\frac{5}{7}$ n.

11. How much is in $\frac{4}{5}$ of a shilling? Ans. 9d. $2\frac{2}{5}$ qrs.

12. How much is in $\frac{12}{16}$ of a lb. Troy? Ans. 9 oz.

13. How much is in $\frac{9}{13}$ of a day?
Ans. 16 h. 36 m. $55\frac{5}{13}$ s.

14. How much is in $\frac{10}{64}\frac{7}{0}$ of a lb. apothecary?
Ans. 2 oz. 3 grs.

15. How much is in $\frac{19}{1}\frac{7}{9}\frac{7}{2}$ of a cwt.? Ans. 12 lb. 5 oz.

16. How much is in $\frac{9}{16}$ of a day? Ans. 13 h. 30 m.

17. How much is $\frac{4}{5}$ of a pound Troy?
Ans. 9 oz. 12 dwt.

18. What is equal to $\frac{7}{13}$ of a day?
Ans. 12 h. 55 m. $23\frac{1}{13}$ sec.

19. Reduce $\frac{27}{15}\frac{7}{3}$ of a ton to its proper quantity.
Ans. 3 cwt. 1 qr. 18 lb. 12 oz. $8\frac{7}{16}$ dr.

20. Reduce $\frac{4}{7}$ of a mile to its proper quantity.
Ans. 182 rods. 4 yds. 2 ft. $1\frac{1}{7}$ in.

21. Add together $\frac{5}{8}$ of pound and $\frac{3}{8}$ of a shilling.
£$\frac{5}{8}$ = 11s. 1d. $1\frac{1}{2}$qrs. $\frac{3}{8}$s. = 4d. 2 qrs. Ans. 11s. 5d. $3\frac{1}{2}$ qrs.

22. Add together $\frac{1}{4}$ of a week, $\frac{1}{2}$ of a day, and $\frac{1}{5}$ of an hour. Ans. 2 days, 2 h. 12 m.

23. Add together $\frac{3}{10}$ cwt. $\frac{5}{7}$ of a qr. and $\frac{9}{11}$ of a pound.
Ans. 1 qr. 13 lb. 13 oz. $1\frac{5}{11}$ dr.

24. What is the sum of $\frac{3}{8}$ of a lb. Troy, $\frac{8}{9}$ of an ounce, and $\frac{7}{10}$ of a pennyweight? Ans. 5 oz. 8 dwt. $11\frac{7}{15}$ grs.

25. What is the sum of $\frac{2}{7}$ of £15; £$3\frac{3}{7}$; $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{4}$ £.; and $\frac{2}{7}$ of $\frac{3}{7}$ of a shilling? Ans. £7, 17s. $5\frac{4}{7}$d.

REDUCTION OF FRACTIONS OF ONE DENOMINATION TO FRACTIONS OF ANOTHER DENOMINATION.

THIS may be done by multiplying or dividing the fractions in the same manner as if they were whole numbers.

1. What is $\frac{3}{854}$ of a £ when reduced to the fraction of a farthing?

Ans. $\frac{3}{854}$ *of a £ is* $= \frac{3}{854}$ *of* 20*s.* $\frac{1}{854}$ *of* 20*s.* $= \frac{20}{854}$*s., and* $\frac{3}{854}$ *of* 20*s. is* 3 *times* $\frac{20}{854}$*s. which equals* $\frac{60}{854}$*s.* $\frac{60}{854}$*s.* $= \frac{60}{854}$ *of* 12*d.* $\frac{1}{854}$ *of* 12*d.* $= \frac{12}{854}$ *of a penny, and* $\frac{60}{854}$ *of* 12*d. is* 60 *times* $\frac{12}{854}$*d., which equals* $\frac{720}{854}$*d.* $\frac{720}{854}$*d.* $= \frac{720}{854}$ *of* 4 *qrs.* $\frac{1}{854}$ *of* 4 *qrs.* $= \frac{4}{854}$ *of a qr., and* $\frac{720}{854}$ *qr.* $= 720$ *times* $\frac{4}{854}$ *qrs. which equals* $\frac{2880}{854}$ *qr.*

3 = numerator.
20
60
12
720
4
2880

Ans. $\frac{2880}{854}$ *qr.* $= \frac{1440}{427}$

or, $\frac{3 \times 20 \times 12 \times 4}{854} = \frac{2880}{854}$.

2. In $\frac{1}{6}$ of a yard, what part of a quarter?

Ans. $\frac{1}{6} \times 4 = \frac{4}{6} = \frac{2}{3}$ qr.

3. Reduce $\frac{1}{96}$ of a shilling to the fraction of a farthing.

Ans. $\frac{48}{96}$ qr. $= \frac{1}{2}$ qr.

4. Reduce $\frac{1}{1\,2\,6}$ of a cwt. to the fraction of a lb.

Ans. $\frac{8}{9}$ lb.

5. Reduce $\frac{1}{2\,2\,4}$ of a week to the fraction of an hour.

Ans. $\frac{3}{4}$ hour.

6. Reduce $\frac{1}{1\,6\,0\,0}$ of a day to the fraction of a minute.

Ans. $\frac{9}{10}$ m.

7. Reduce $\frac{1}{2\,1\,1\,2}$ of a mile to the fraction of a yard.

Ans. $\frac{5}{8}$ yd.

8. Reduce $\frac{7}{2\,6\,0\,0}$ of a year to the fraction of a day.

Ans. $\frac{5\,1\,1}{5\,2\,0}$ day.

9. Reduce $\frac{9}{3\,4\,3}$ of a gallon to the fraction of a gill.

Ans. $\frac{1\,8}{1\,9}$ gl.

10. Reduce $\frac{5}{3\,3\,5\,0\,0\,0}$ of an acre to the fraction of a sq. ft.

Ans. $\frac{1\,2\,1}{1\,2\,5}$ ft.

11. Reduce $\frac{5}{8}$ of a penny to the fraction of a £.

Ans. *$\frac{5}{8}$ of a penny equals $\frac{5}{8}$ of $\frac{1}{12}$ of a shilling, which = $\frac{5}{96}$ of a shilling.* $\frac{5}{96}$ *of a shilling = $\frac{5}{96}$ of $\frac{1}{20}$ of a pound, which = $\frac{5}{1440}$£; which = $\frac{1}{288}$£, the answer.*

$\frac{5}{8}$ of $\frac{1}{12} = \frac{5}{96}$
$\frac{5}{96}$ of $\frac{1}{20} = \frac{5}{1920}$
$\frac{5}{1920} = \frac{1}{288}$.

12. Reduce $\frac{10}{11}$ of a minute to the fraction of a day.
Ans. $\frac{5}{1584}$ day.

13. Reduce $\frac{6}{7}$ of a lb. to the fraction of a cwt.
Ans. $\frac{3}{28}$ cwt.

14. Reduce $\frac{1}{3}$ of a nail to the fraction of a yard.
Ans. $\frac{1}{12}$ yd.

15. Reduce $\frac{6}{11}$ of a gill to the fraction of a gallon.
Ans. $\frac{3}{176}$ gal.

16. Reduce $\frac{9}{14}$ of a foot to the fraction of a mile.
Ans. $\frac{3}{2464}$ mile.

17. Reduce $\frac{7}{13}$ of a grain to the fraction of a lb.
Ans. $\frac{7}{8580}$ lb.

18. Reduce $\frac{5}{9}$ of an ounce to the fraction of a C.
Ans. $\frac{1}{3528}$ C.

19. Reduce $\frac{6}{7}$ of an oz. to the fraction of a cwt.
Ans. $\frac{3}{3136}$ cwt.

20. Reduce $\frac{4}{7}$ of a sq. yd. to the fraction of an acre.
Ans. $\frac{1}{8470}$ acre.

21. Reduce $\frac{7}{8}$ of a nail to the fraction of a yard.
Ans. $\frac{7}{144}$ yd.

22. Reduce $\frac{3}{4}$ of a farthing to the fraction of a £.
Ans. $\frac{1}{1440}$£.

23. Reduce $\frac{8}{11}\frac{8}{12}$ of a gill to the fraction of a gallon.
Ans. $\frac{1}{32}$ gallon.

24. Reduce $\frac{4}{3}\frac{4}{7}\frac{7}{8}\frac{8}{0}$ of a mile to the fraction of a foot.
Ans. $\frac{2}{3}\frac{1}{4}\frac{1}{7}\frac{2}{5}$ ft.

25. Reduce $\frac{11}{14}$ of a £ to the fraction of a farthing.
Ans. $\frac{5}{7}\frac{2}{1}\frac{8}{}\frac{0}{}$ qrs.

DUODECIMALS.

1	2	3	4	5	6
2					
3					
4					
5					
6					
7					

By looking at the opposite diagram, it will be seen that a surface 6 inches long and 1 inch wide, contains 6 *square inches;* and that if there are 7 of these surfaces, they all contain 7 times 6 square inches, or 42 square inches. Whence we see that *the area of a surface is found by multiplying the length by the breadth.*

When these dimensions are expressed in feet and inches, the operation is performed by *duodecimals* or *twelfth parts*, according to the following table.

$\frac{1}{12}$ of a foot = 1′, or 1 prime.

$\frac{1}{12}$ of a prime = 1″, or 1 second, which = $\frac{1}{12}$ of $\frac{1}{12}$ of 1 ft. or $\frac{1}{144}$ of 1 ft.

$\frac{1}{12}$ of a second = 1‴, or 1 third, which = $\frac{1}{12}$ of $\frac{1}{144}$ of 1 ft. or $\frac{1}{1728}$ of 1 ft. &c. &c.

A prime may stand for 1 linear inch, or 12 square inches, or 144 cubic inches, &c.

From this table, it is easily seen that *feet*, × *primes* ($\frac{1}{12}$), produces *primes; primes* × *primes*, produces *seconds* ($\frac{1}{144}$); *primes* × *seconds*, produces *thirds*, ($\frac{1}{1728}$); and generally, that the denomination of the product is always denoted by the sum of the indices. Thus ″ × ‴ = ⁗.

Example.

Multiply 6 ft. 8 in. by 5 ft. 7 in.

Ans. $\frac{7}{12}$ of $\frac{8}{12} = \frac{56}{144} = (\frac{48}{144} + \frac{8}{144}) = \frac{4}{12} + \frac{8}{144}$. $\frac{7}{12}$ of 6 = $\frac{42}{12}$, which with $\frac{4}{12} = \frac{46}{12} = 3 + \frac{10}{12}$. 5 times $\frac{8}{12} = \frac{40}{12} = 3 + \frac{4}{12}$. 5 times 6 ft. = 30 ft., which + 3 ft. = 33 ft. The products added together equal 37 ft. 2′ 8″. The $\frac{2}{3}$ of a sq. foot = 24 sq. in., which with the $\frac{8}{144}$ = 32 sq. in.

	6 ft.	8′
	5	7′
3	10′	8″
33	4′	
37 ft.	2′	8″

RULE.

Place the multiplier under the multiplicand, so that primes shall be under primes, seconds under seconds, &c.

Beginning on the right hand of the multiplier, multiply the multiplicand as in compound multiplication; taking care to carry one for every 12, and denoting the quantity set down, by as many accents as are in *both the terms* multiplied.

Then proceed in the same manner with the next right hand term of the multiplier, and then with the next, &c.; taking care to place the first number in each partial product, exactly under the multiplying figure. The sum of the partial products will be the answer.

SUMS FOR EXERCISE.

1. Multiply 5 ft. 3 in. by 4 ft. 6 in.
Ans. 23 ft. 7′ 6″ = 23 sq. ft. 90 sq. in.

2. Multiply 8 ft. 11 in. by 7 ft. 10 in.
Ans. 69 ft. 10′ 2″ = 69 sq. ft. 122 sq. in.

3. Multiply 5 ft. 10 in. by 4 ft. 7 in.
Ans. 26 ft. 8′ 10″ = 26 sq. ft. 106 sq. in.

4. Required the contents of a board that measures 28 ft. 10 in. long, and 3 ft. 5 in. wide. Ans. 98 sq. ft. 74 sq. in.

5. A floor is 79 ft. 8 in. by 8 ft. 11 in. How many square feet are there in it? Ans. 710 sq. ft. 52 sq. in.

6. What is the area of a lot 82 ft. 6 in. by 13 ft. 3 in.?
Ans. 1093 sq. ft. 18 sq. in.

7. How many solid feet in a load of wood 8 ft. 4 in. long, 2 ft. 6 in. high, and 3 ft. 3 in. wide?

☞ Multiply the length by the breadth, and the product by the thickness. Ans. 67 ft. 8′ 6″.

8. How many solid feet are there in a stick of timber 70 ft. long, 1 ft. 3 in. thick, and 1 ft. 6 in. wide?
Ans. 131 ft. 3′ = 131 sol. ft. 432 in.

9. How many solid ft. in a pile of wood 6 ft. 7 in. long, 3 ft. 5 in. high, and 3 ft. 8 in. wide?
Ans. 82 sol. ft. 820 sol. in.

CONVERSION OF TERMS.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. A MAN divided some money among 4 paupers, giving each of them 4 cents. What part of the whole money did he give to each? Of how much money is 4 cents the $\frac{1}{4}$?

2. A teacher divided a bunch of quills among ten pupils, giving each of them 3. What part of the whole bunch did he give to each? Of what number is 3 the $\frac{1}{10}$? Ans. 3 is the $\frac{1}{10}$ of 10 times 3, which is 30?

3. Of what number is 2 the $\frac{1}{4}$? $\frac{1}{7}$? $\frac{1}{8}$?

4. Of what number is 4 the $\frac{1}{3}$? $\frac{1}{7}$? $\frac{1}{9}$?

5. Of what number is 5 the $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{8}$?

6. Of what number is 7 the $\frac{1}{3}$? $\frac{1}{6}$? $\frac{1}{9}$?

7. Of what number is 9 the $\frac{1}{7}$? $\frac{1}{5}$? $\frac{1}{8}$?

8. Of what number is $\frac{1}{2}$ the half? Third? Fourth? Fifth? Sixth? Seventh? Eighth? Ninth?

9. Of what is $\frac{1}{3}$ the $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$?

10. Of what is $\frac{2}{9}$ the $\frac{1}{3}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$?

11. Of what is $\frac{3}{4}$ the $\frac{1}{2}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{10}$?

12. Of what is $\frac{5}{8}$ the $\frac{1}{3}$? $\frac{1}{2}$? $\frac{1}{4}$? $\frac{1}{7}$?

13. Of what is $\frac{4}{7}$ the $\frac{1}{4}$? $\frac{1}{9}$? $\frac{1}{5}$? $\frac{1}{10}$?

14. Of what is $4\frac{5}{6}$ the $\frac{1}{2}$? $\frac{1}{4}$? $\frac{1}{7}$? $\frac{1}{8}$?

15. If 9 is $\frac{3}{5}$ of some number, what is that number? Ans. *If* 9 *is* $\frac{3}{5}$ *of some number*, the $\frac{1}{5}$ of 9, which is 3, must be *the* $\frac{1}{5}$ *of that number. And* 3 *is the* $\frac{1}{5}$ *of* 5 *times* 3, *which is* 15.

16. If 6 is $\frac{3}{8}$ of some number, what is that number?

17. Of what number is 6 the $\frac{3}{7}$? $\frac{3}{10}$?

18. Of what number is 4 the $\frac{2}{7}$? $\frac{2}{5}$?

19. Of what number is 10 the $\frac{2}{3}$? $\frac{5}{8}$?

20. Of what number is 2 the $\frac{3}{4}$? Ans. *If 2 is $\frac{3}{4}$ of some number, the $\frac{1}{3}$ of 2, which is $\frac{2}{3}$, is $\frac{1}{4}$ of that number. And $\frac{2}{3}$ is $\frac{1}{4}$ of 4 times $\frac{2}{3}$, which is $\frac{8}{3}$, or $2\frac{2}{3}$?*

21. Of what number is 3 the $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{6}$?

22. Of what number is 5 the $\frac{3}{5}$? $\frac{3}{7}$? $\frac{4}{9}$?

23. Of what number is 9 the $\frac{4}{5}$? $\frac{2}{8}$? $\frac{6}{10}$?

INDUCTIVE EXERCISES FOR THE SLATE.

1. A CHILD that was 6 years of age was told that his age was $\frac{1}{8}$ of his father's. What was his father's age?

Ans. As the father's age was $\frac{8}{8}$ of itself, and the son's age was only $\frac{1}{8}$ of it, the father must have been 8 times as old as the son; which is 48 years.

2. If 25 is the $\frac{1}{14}$ of a certain number, what is that number? Ans. 14 times 25, which = 350.

3. If a man, by giving away 47 apples, gives $\frac{1}{9}$ of what he has, how many has he? Ans. 423.

4. Of what number is 36 the $\frac{1}{12}$? Ans. 432.

5. A man who owns $\frac{1}{3}$ of a certain ship, values his share of her at $8375. What is the worth of the whole ship at that rate? Ans. $41875.

6. A told B that he was 18 years old. Then, replied B, your age is $\frac{2}{7}$ of mine. How old was B?

Ans. If 18 years was $\frac{2}{7}$ of B's age, then $\frac{1}{2}$ of 18 years, which is 9 years, must be $\frac{1}{7}$ of his age; and, of course, 7 times 9 years, which is 63 years, is his age.

7. If 16 is $\frac{4}{9}$ of some number, what is that number?
Ans. 36.

8. If 21 is $\frac{7}{10}$ of some number, what is that number?
Ans. 30.

9. Of what number is 28 the $\frac{4}{11}$? Ans. 77.

10. Of what number is 56 the $\frac{8}{15}$? Ans. 105.

11. Of what number is 38 the $\frac{19}{20}$? Ans. 40.

12. Of what number is 45 the $\frac{9}{20}$? Ans. 100.

13. Of what number is 108 the $\frac{12}{35}$? Ans. 315.

14. Of what number is 231 the $\frac{77}{400}$? Ans. 1200.

15. A teacher once said that $\frac{3}{7}$ of his school were studying arithmetic. It was found that he had 27 arithmeticians. Of how many did his school consist?

Ans. If $\frac{3}{7}$ of his school was 27, the $\frac{1}{7}$ of his school was $\frac{1}{3}$ of 27. But if $\frac{1}{7}$ of the school was $\frac{1}{3}$ of 27, $\frac{7}{7}$ of it, or the whole school, must be 7 times $\frac{1}{3}$ of 27, which is $\frac{7}{3}$ of 27.

Principle.—From the last answer, it appears that if the arithmeticians were $\frac{3}{7}$ of the school, then the school was equal to $\frac{7}{3}$ of the arithmeticians; and that the answer is found by merely inverting the terms of the fraction. So, also, if one number is $\frac{4}{9}$ of another, the last number is $\frac{9}{4}$ of the first.

16. If a cargo is worth $\frac{8}{5}$ of the ship which carries it, what part of the cargo is the ship worth? Ans. $\frac{5}{8}$.

17. If 16 is $\frac{4}{11}$ of some number, what is that number?
Ans. The number is $\frac{11}{4}$ of 16, which = 44.

18. If 96 is $\frac{8}{15}$ of some number, what is that number?
Ans. 180.

19. If 144 is $\frac{12}{23}$ of some number, what is that number?
Ans. 276.

20. If 351 is $\frac{39}{50}$ of some number, what is that number?
Ans. 450.

21. If 594 is $\frac{37}{46}$ of some number, what is that number?
Ans. $738\frac{18}{37}$

DEFINITION

A conversion of terms is made by supposing that the statement is the *answer* of some proposition; and that we use that answer to discover what that proposition was.

RULE.

First, Consider the number by which the supposed answer was obtained from the required number, as a *multiplying fraction.*

Second, Invert the terms of the multiplying fraction, and then proceed as in multiplication of fractions.

NOTE.—If the supposed answer was obtained by multiplying by an integer, that integer may be considered as a fraction, with 1 as its denominator. Thus, if it was 7 times some number, it was $\frac{7}{1}$ of that number.

APPLICATION OF THE RULE.

1. If $\frac{3}{4}$ of a yard cost 5 dollars, will 1 yard cost?
1 *yard will cost* $\frac{3}{4}$ *of* $5., &c. Ans. $\$7\frac{1}{3}$.

2. If $\frac{7}{9}$ of a yard cost $3\frac{3}{4}$ dollars, what will 1 yard cost?
Ans. $\$4\frac{19}{28}$.

3. If $\frac{4}{5}$ of a barrel of flour cost $3\frac{1}{7}$ dollars, what will 1 barrel cost? Ans. $\$7\frac{1}{14}$.

4. If $\frac{7}{10}$ of a ton of coal cost $3\frac{1}{9}$ dollars, what is the cost of a ton? Ans. $\$10\frac{10}{63}$.

5. If $2\frac{3}{4}$ yards of calico is worth $\frac{7}{16}$ of a dollar, what is 1 yard worth? Ans. $\$\frac{7}{55}$.

6. If $3\frac{1}{4}$ pounds of raisins cost $\frac{6}{13}$ of a dollar, what is the cost of 1 lb.? $7\frac{1}{2}$ lbs.? Ans. $\$\frac{12}{91}$, and $\$\frac{12}{13}$.

7. If $\frac{4}{9}$ of a pound of raisins cost $\frac{3}{4}$ of a shilling, what is the cost of 1 lb.? $5\frac{4}{11}$ lbs.? Ans. $1\frac{11}{16}$ s., and $9\frac{9}{176}$ s.

8. If $18\frac{3}{4}$ lbs. of butter cost $\$3\frac{7}{8}$, what is the cost of 1 lb.? $16\frac{1}{7}$ lbs. Ans. $\$\frac{170}{877}$, and $\$3\frac{2143}{3625}$.

9. If $\frac{4}{5}$ of a pound of sugar cost $\frac{7}{11}$ of a dollar, what will $3\frac{7}{9}$ lbs. cost? Ans. $\$\frac{595}{198}$

10. 89 is $\frac{4}{11}$ of what number? Ans. $244\frac{3}{4}$.

11. $\frac{98}{1101}$ is $\frac{11}{27}$ of what number? Ans. $\frac{864}{40637}$.

12. $\frac{21}{30}$ is $\frac{7}{15}$ of what number? Ans. $1\frac{2}{13}$.

13. Of what number is $41\frac{3}{7}$ the $\frac{42}{45}$? Ans. $42\frac{7}{24}$.

14. Of what number is $308\frac{4}{7}$ the $\frac{5}{12}$? Ans. $375\frac{15}{110}$.

15. 107 is $\frac{65}{102}$ of what number? Ans. $167\frac{59}{65}$.

16. $496\frac{9}{46}$ is $\frac{71}{96}$ of what number? Ans. $670\frac{1400}{1633}$.

17. $195\frac{19}{31}$ is the $\frac{45}{365}$ of what number? Ans. $1584\frac{316}{350}$.

18. A person, after spending $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{7}$ of his money, had 130 dollars left. How much had he at first?

Ans. $600.

19. In a certain school, $\frac{1}{2}$ of the scholars belong to the first class, $\frac{2}{7}$ of them to the second, $\frac{1}{8}$ of them to the third, and the rest of them to the fourth. What part of them belonged to the fourth? And supposing there are five scholars in the fourth, how many are there in the whole school?

Ans. 56.

20. Of a certain sum of money, A had $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $26. What part had D; and how much was the whole sum?

Ans. D had $\frac{13}{60}$ of the whole; and the whole was $120.

21. Jane and Amelia had each a certain sum of money. Jane spends $\frac{1}{3}$ of hers during the first year, and Amelia spends $\frac{1}{5}$ of hers in the same time. They then find that Jane has 400 dollars more on hand than Amelia. What was the sum that each had at first? Ans. $900.

22. There are two numbers whose difference is 40. One number is $\frac{5}{6}$ of the other. What are the numbers?

Ans. 200, and 240.

23. What number is that, a sixth part of which exceeds an eighth part of it by 20? Ans. 480.

24. Divide 2000 dollars into two parts, of which the less shall be $\frac{7}{9}$ of the greater. Ans. 1125, and 875.

25. What sum of money is that whose $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, added together, amount to 94 dollars? Ans. $120.

DIVISION BY VULGAR FRACTIONS.

EXERCISES FOR THOSE WHO HAVE NOT STUDIED MENTAL ARITHMETIC.

1. How many $\frac{1}{2}$ are there in 2? 3? 5? 6? $6\frac{1}{2}$? $7\frac{1}{2}$?

2. How many $\frac{1}{3}$ are there in 2? 4? $4\frac{1}{3}$? $5\frac{2}{3}$? $7\frac{1}{3}$? $8\frac{2}{3}$?

3. How many $\frac{1}{4}$ are there in 1? $3\frac{3}{4}$? $7\frac{1}{4}$? $9\frac{3}{4}$? $10\frac{1}{4}$?

4. How many times $\frac{2}{3}$ are there in 4, or $\frac{12}{3}$? In 6, or $\frac{18}{3}$? In $3\frac{1}{3}$, or $\frac{10}{3}$? In $4\frac{2}{3}$, or $\frac{14}{3}$?

5. How many times $\frac{3}{4}$ are there in 3, or $\frac{12}{4}$? $2\frac{1}{4}$, or $\frac{9}{4}$? $4\frac{3}{4}$, or $\frac{18}{4}$? $5\frac{1}{4}$, or $\frac{21}{4}$?

6. How many times $\frac{2}{5}$ are there in $3\frac{1}{5}$? Ans. $3\frac{1}{5}$ equals $\frac{16}{5}$; *therefore, the question is, how many times are there* $\frac{2}{5}$ *in* $\frac{16}{5}$? *Eight times.*

7. How many times are there $\frac{3}{7}$ in $3\frac{1}{7}$? 4? $5\frac{1}{7}$? 6? $8\frac{2}{7}$?

8. How many times are $\frac{3}{5}$ contained in $3\frac{3}{5}$? $4\frac{1}{5}$? 6? $7\frac{1}{5}$?

9. How many times are $\frac{5}{6}$ contained in $1\frac{4}{6}$? $3\frac{2}{6}$? 5? $6\frac{4}{6}$?

10. How many times are $\frac{2}{7}$ contained in $2\frac{0}{7}$? $3\frac{5}{7}$? $4\frac{4}{7}$? $7\frac{3}{7}$? $7\frac{1}{7}$? $8\frac{2}{7}$?

11. How many times 3 apples in 4 apples? Then how many times 3 fifths in four fifths? How many times $\frac{3}{7}$ in $\frac{4}{7}$? $\frac{3}{5}$ in $\frac{4}{5}$?

12. How many times $\frac{3}{5}$ in $\frac{4}{7}$? In $1\frac{2}{5}$? $3\frac{4}{5}$? $7\frac{2}{5}$? $7\frac{4}{5}$?

13. How many times $\frac{2}{7}$ in $\frac{3}{7}$? $\frac{4}{7}$? 1? $1\frac{4}{7}$? 5?

14. How many $\frac{3}{4}$ in $\frac{1}{4}$? $1\frac{1}{4}$? $1\frac{3}{4}$? 2? $2\frac{1}{4}$? $3\frac{1}{4}$?

15. How many $\frac{2}{3}$ in $\frac{1}{3}$? 1? $2\frac{1}{3}$? $3\frac{2}{3}$? 5? $5\frac{2}{3}$?

16. How many times $\frac{3}{8}$ in $\frac{4}{8}$? In $\frac{7}{8}$? $1\frac{3}{8}$? $\frac{1}{8}$? 2? $2\frac{7}{8}$? $3\frac{4}{8}$?

The pupil should be told that we cannot divide one kind of things by another kind of things. Thus, we cannot say, *how many times* 3 *apples in* 12 *pears; nor* 2 *cents in* 3 *marbles.* Neither can we say, how many times 2 thirds in 3 fourths; nor how many times 4 fifths in 20 fourths; but we must bring them to the same name; that is, we must first reduce them to a common denominator.

17. How many times $\frac{2}{12}$ are there in $\frac{2}{3}$? Ans. $\frac{8}{12}$ *are equal to* $\frac{12}{18}$? *Therefore, the question is, How many times* $\frac{2}{12}$ *in* $\frac{12}{18}$? *Six times.*

18. How many $\frac{3}{4}$ in $\frac{6}{8}$? In $\frac{7}{8}$? $\frac{3}{8}$? $\frac{9}{12}$? $\frac{7}{12}$? $\frac{12}{16}$? $\frac{11}{16}$?

19. How many times $\frac{2}{5}$ are contained in $\frac{4}{10}$? In $\frac{7}{10}$? $\frac{6}{15}$? $\frac{12}{15}$? $\frac{18}{20}$? $\frac{13}{20}$? $\frac{17}{20}$?

20. How many times are $\frac{3}{5}$ contained in $\frac{1}{10}$? In $\frac{4}{10}$? $\frac{11}{15}$? $\frac{14}{15}$? $\frac{13}{20}$? $\frac{16}{20}$?

21. How many times $\frac{6}{8}$ in $\frac{1}{12}$? In $\frac{14}{18}$? $\frac{3}{24}$? $\frac{5}{30}$? $\frac{27}{30}$?

22. How many times $\frac{3}{16}$ are there in $\frac{5}{8}$? In $\frac{2}{5}$? $\frac{2}{3}$? $\frac{1}{3}$?

23. How many times $\frac{4}{24}$ are there in $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{7}{12}$?

24. How many times $\frac{3}{7}$ in $\frac{3}{14}$? In $\frac{15}{14}$? In $\frac{6}{28}$? In $\frac{38}{42}$?

25. How many times $\frac{5}{8}$ in $\frac{3}{16}$? $\frac{15}{24}$? $\frac{24}{32}$? $\frac{17}{40}$? $\frac{47}{48}$? $\frac{47}{56}$?

26. How many times $\frac{9}{7}$ in $\frac{18}{5}$? $1\frac{4}{5}$ in $\frac{18}{5}$? $1\frac{4}{5}$ in $3\frac{2}{3}$?

27. How many times $\frac{7}{4}$ in $\frac{18}{4}$? $1\frac{3}{4}$ in $4\frac{3}{4}$? In $5\frac{1}{4}$? $7\frac{3}{4}$?

28. How many times $\frac{5}{7}$ in $4\frac{3}{7}$? $1\frac{3}{7}$ in $4\frac{3}{7}$? $1\frac{1}{3}$? $2\frac{1}{3}$? $3\frac{3}{7}$?

29. How many times $1\frac{3}{4}$ in $1\frac{1}{4}$? $2\frac{3}{4}$? $4\frac{1}{4}$? $5\frac{3}{4}$? $6\frac{1}{4}$?

30. How many times $2\frac{3}{7}$ in $6\frac{1}{7}$? $3\frac{1}{7}$? $1\frac{1}{7}$? $5\frac{3}{7}$? $4\frac{6}{7}$?

31. How many times $3\frac{5}{8}$ in $1\frac{1}{8}$? $2\frac{7}{8}$? $4\frac{6}{8}$? $\frac{3}{8}$? $\frac{7}{8}$? $\frac{1}{8}$?

32. How many times is $\frac{1}{6}$ contained in $\frac{1}{4}$? Ans. $\frac{1}{6}$ *equals* $\frac{2}{12}$; *and* $\frac{1}{4}$ *equals* $\frac{3}{12}$. $\frac{2}{12}$ *is contained in* $\frac{3}{12}$ *one and a half times.*

33. How many times is $\frac{1}{2}$ contained in $\frac{1}{3}$? $2\frac{1}{2}$?

34. How many times is $\frac{1}{3}$ contained in $\frac{1}{2}$? $1\frac{2}{3}$?

35. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$? $2\frac{1}{4}$?

36. How many times is $\frac{3}{4}$ contained in $\frac{1}{6}$? $1\frac{2}{3}$?

37. How many times is $\frac{3}{7}$ contained in $1\frac{2}{3}$? $3\frac{1}{8}$?

INDUCTIVE EXERCISES FOR THE SLATE.

1. If 1 penknife cost $\frac{2}{5}$ of a dollar, how many penknives can be bought for 4 dollars?

Ans. In examples of this kind, we must apply the principle that was adopted on page 140; that is, we must change 4 dollars to *fifths* of a dollar. Thus, 4 dollars $= \frac{20}{5}$ of a dollar. Therefore, we can buy as many penknives as there are $\frac{2}{5}$ of a dollar in $\frac{20}{5}$ of a dollar. The answer is 10.

2. How many times is $\frac{2}{5}$ contained in 6?

3. How many times is $\frac{3}{4}$ contained in 6?

4. How many times is $\frac{4}{7}$ contained in 8?

5. How many times is $\frac{5}{8}$ contained in 14? Ans. $22\frac{7}{5}$.

6. How many times is $\frac{7}{10}$ contained in 18? Ans. $25\frac{5}{7}$.

7. How many times is $1\frac{4}{5}$ contained in 41? Ans. $22\frac{7}{9}$.

8. How many times is $2\frac{3}{4}$ contained in 37? Ans. $13\frac{7}{8}$.

9. How many times is $4\frac{4}{7}$ contained in $22\frac{2}{7}$? Ans. $4\frac{28}{36}$.

10. How many times is $8\frac{3}{5}$ contained in $7\frac{1}{3}$? Ans. $\frac{36}{43}$.

11. How many times is $\frac{3}{4}$ contained in $\frac{11}{12}$?

Ans. In this sum, we must bring the fractions to a common denominator. $\frac{3}{4} = \frac{9}{12}$; *and* $\frac{9}{12}$ *is contained in* $\frac{11}{12}$, *one and two-ninths times.* $\frac{11}{12} \div \frac{9}{12} = 1\frac{2}{9}$.

12. How many times is $\frac{3}{5}$ contained in $\frac{18}{20}$? Ans. $1\frac{1}{2}$.
13. How many times is $\frac{4}{7}$ contained in $1\frac{3}{14}$? Ans. $1\frac{5}{8}$.
14. How many times is $\frac{4}{9}$ contained in $1\frac{3}{7}$? Ans. $3\frac{3}{14}$.
15. How many times is $\frac{6}{7}$ contained in $4\frac{1}{3}$? Ans. $5\frac{1}{18}$.
16. How many times is $\frac{4}{13}$ contained in $2\frac{1}{9}$? Ans. $6\frac{31}{36}$.
17. How many times is $1\frac{7}{8}$ contained in $4\frac{3}{5}$? Ans. $2\frac{34}{75}$.

It may be well now to see the manner in which the foregoing sums have been performed. Suppose we wish to know how many times $\frac{3}{20}$ are contained in $\frac{7}{8}$. We first bring them to a common denominator, which is 160; and then say $\frac{24}{160}$ are contained in $\frac{140}{160}$, $5\frac{20}{24}$ times. Now, by examining the operation, it will be seen that we do not use the common denominator at all. We may therefore save ourselves the trouble of multiplying the two denominators 8 and 20. The operation will then be as in the margin. *We multiply the numerator of the dividend by the denominator of the divisor, to obtain the new dividend. And we multiply the denominator of the dividend by the numerator of the divisor, for the new divisor. And we then divide by putting the divisor under the dividend.*

$\frac{7}{8} \div \frac{3}{20} = \frac{140}{160} \div \frac{24}{160}$

$24)140(5\frac{20}{24}$
120
20

$\frac{7}{8} \div \frac{3}{20} = \frac{140}{160} \div \frac{24}{160} = \frac{140}{24} = 5\frac{20}{24} \div 5\frac{5}{6}$.

$\frac{7}{8} \div \frac{3}{20} = \frac{140}{24}$.
$\frac{140}{24} = 5\frac{20}{24} = 5\frac{5}{6}$.

The pupil may perform the following in the same manner:

18. How many times is $\frac{4}{9}$ contained in $\frac{12}{17}$? Ans. $1\frac{19}{17}$.
19. How many times is $\frac{2}{7}$ contained in $\frac{14}{15}$? Ans. $3\frac{4}{15}$.
20. How many times is $\frac{4}{13}$ contained in $\frac{27}{45}$? Ans. $2\frac{121}{180}$.
21. How many times is $\frac{7}{29}$ contained in $\frac{14}{31}$? Ans. $1\frac{27}{31}$.
22. How many times is $\frac{97}{100}$ contained in $\frac{14}{15}$? Ans. $\frac{280}{291}$.

23. How many times is $\frac{131}{143}$ contained in $\frac{4}{7}$? Ans. $\frac{572}{917}$.

24. How many times is $\frac{147}{156}$ contained in $\frac{5}{6}$? Ans. $\frac{130}{147}$.

Rule.

First, If the divisor and the dividend have a common denominator, divide the numerator of the dividend by the numerator of the divisor, as in whole numbers.

Second, If either the divisor or the dividend be a whole number, reduce it to a fraction of the same denominator with the other, and then divide the numerator of the dividend by the numerator of the divisor.

Third, When the divisor and the dividend have not a common denominator, multiply the numerator of the dividend by the denominator of the divisor for a new numerator; and multiply the denominator of the dividend by the numerator of the divisor for a new denominator.

Application of the Rule.

1. If 1 bushel of potatoes cost $\frac{1}{4}$ of a dollar, how many bushels will cost $\frac{3}{4}$ of a dollar?

2. If 1 bushel of apples cost $\frac{1}{8}$ of a dollar, how many bushels can be bought for $2\frac{1}{8}$ dollars?

3. If one yard of calico cost $\frac{1}{5}$ of a dollar, how many yards can be bought for $3\frac{3}{5}$ dollars?

4. At $\frac{2}{3}$ of a dollar a yard, how many yards can be bought for $\frac{7}{8}$ of a dollar?

5. At $1\frac{3}{5}$ of a dollar a pound, how much tea can be bought for $15? Ans. $9\frac{3}{8}$ lbs.

6. At $\frac{1}{8}$ of a dollar a pound, how much sugar can be bought for $3\frac{1}{4}$ dollars? Ans. $30\frac{2}{5}$ lbs.

7. At $\frac{1}{5}$ of a dollar a bushel, how many bushels of apples can be bought for $6\frac{2}{3}$ dollars? Ans. $31\frac{1}{3}$ bush.

8. At $\frac{9}{10}$ of a dollar a bushel, how many bushels of corn can be bought for $8\frac{7}{10}$ dollars?

Ans. *As many bushels, &c.*

$8\frac{7}{10} = \frac{87}{10}$ *which* $\div \frac{9}{10} = \frac{1392}{90}$, *which* $= \frac{232}{15}$, *which* $= 15\frac{7}{15}$.

$8\frac{7}{10} = \frac{87}{10}$.

$\frac{87}{10} \div \frac{9}{10} = \frac{1392}{90} = \frac{232}{15}$

$= 15\frac{7}{15}$.

9. If it takes $1\frac{3}{4}$ yard for a coat, how many coats may be made from $16\frac{1}{2}$ yards? Ans. $9\frac{3}{7}$ coats.

10. At $6\frac{5}{9}$ of a dollar a ton, how many tons of coal may be bought for $72\frac{4}{11}$? Ans. $11\frac{25}{649}$ tons.

11. At 3 dollars a barrel, what part of a barrel of cider can be bought for $\frac{3}{7}$ of a dollar? [$\frac{3}{7} \div \frac{21}{7}$.] Ans. $\frac{3}{21}$, or $\frac{1}{7}$ bbl.

12. At $5 a yard, what part of a yard of cloth can be bought for $\frac{9}{14}$ of a dollar? Ans. $\frac{9}{70}$ of a yd.

13. At 2 dollars a gallon, how many gallons of wine can be bought for $9\frac{3}{7}$ dollars? Ans. $4\frac{5}{7}$ gals.

14. At $10 a ton, how much hay can be bought for $37\frac{4}{15}$ dollars? Ans. $3\frac{109}{150}$ tons.

15. At 8 cents a pound, how many pounds of sugar can be bought for $47\frac{8}{8}$ cents? Ans. $5\frac{7}{12}$ lbs.

16. At $5\frac{2}{3}$ a barrel, what part of a barrel of flour can be bought for $\frac{6}{7}$ of a dollar? Ans. $\frac{18}{83}$ bbls.

17. At $\frac{3}{4}$ of a dollar a bushel, what part of a bushel can can be bought for $\frac{7}{20}$ of a dollar? Ans. $\frac{7}{15}$ bush.

18. $48\frac{1}{7} \div 4\frac{3}{9} = 11\frac{10}{97}$.

19. $94\frac{9}{16} \div 106\frac{3}{7} = \frac{10591}{11920}$.

20. $47\frac{3}{61} \div \frac{36}{41} = 53\frac{641}{1098}$.

21. $\frac{301}{410} \div 961 = \frac{301}{393610}$.

22. $\frac{41}{366} \div \frac{74}{291} = \frac{3977}{90276}$.

23. $865\frac{4}{11} \div \frac{11}{967} = 76073\frac{40}{121}$.

DECIMAL FRACTIONS.

INDUCTIVE EXERCISES FOR THE SLATE.

1. In the use of vulgar fractions, there is frequently much loss of time, in converting the denominator of one fraction to the denominator of another. On this account, there was invented another kind of fractions, called *Decimal Fractions*, or *Decimals*.

2. The laws of numeration gave rise to this kind of fractions. For, it is evident, that if beginning with 1, the orders *above* it increase by becoming ten times *greater* continually, so we may use orders *below* it that become continually ten times *smaller*.

Thus, we may conceive the unit to be divided into *tenths*, *hundreds*, *thousands*, &c.; so that a *tenth* shall contain ten of the hundredths, a *hundredth* shall contain ten of the thousandths, &c. And, as a figure to the *left* of units expresses *tens*, so one to the *right* of units shall express *tenths*; as a figure in the third order on the left expresses *hundreds*, so one in the third order on the right expresses *hundredths*.

3. But, as we have been accustomed to distinguish the order of *units* by its situation, decimal figures written to the right of it, would lead to some confusion, unless we adopted some sign to show when they commence. On this account, decimals are separated from the integers by a dot. Thus: *One and five tenths* is written 1.5; *Two hundred and four, and four tenths, and seven hundredths*, is written 204.47; &c.

The pupil may now write the following in decimals.

4. Three tenths and four hundredths. Ans. .34.

5. Two tenths, seven hundredths, and six thousandths.

6. Four tenths, two hundredths, and seven thousandths.

7. One tenth, six hundredths, and eight thousandths.

8. Five tenths, nine hundredths, and four thousandths.

9. As 1 tenth is equal to 10 hundredths, how many hundredths is 3 tenths equal to?—How written? Ans. .30.

10. .5 is equal to how many hundredths?

11. .6 is equal to how many hundredths?

12. .6 and .04 are equal to how many hundredths?

13. .4 and .07 are equal to how many hundredths?

14. .9 and .05 are equal to how many hundredths?

Thus, it seems, that .95 may be read either *nine tenths and five hundredths*, or, *ninety-five hundredths*. As the last method is the easiest of enunciation, we shall prefer that. In the same manner, .675 may be read 675 thousandths, because 6 tenths equal $\frac{600}{1000}$; 7 hundredths equal $\frac{70}{1000}$; and these, with the $\frac{5}{1000} = \frac{675}{1000}$.

15. Read .654 both ways, and tell which is the best method.

Ans. $\frac{6}{10}$, $\frac{5}{100}$, and $\frac{4}{1000}$; or $\frac{654}{1000}$; and the last method is the best.

16. Read the following numbers both ways, and tell which is the best.

.37	.676	.439	.456
.3761	.4893	.1607	.3024
.6030	.4700	.6006	.05
.06	.07	.002	.004
.050	.008	.0075	.0064

17. Write the following numbers in the form of decimals.

$\frac{6}{10}$	$\frac{85}{100}$	$\frac{271}{1000}$	$\frac{60}{100}$	$\frac{05}{100}$	$\frac{42}{1000}$
$\frac{7}{10}$	$\frac{33}{100}$	$\frac{365}{1000}$	$\frac{705}{1000}$	$\frac{4}{100}$	$\frac{2}{1000}$
$\frac{37}{100}$	$\frac{94}{100}$	$\frac{438}{1000}$	$\frac{200}{1000}$	$\frac{7}{100}$	$\frac{3}{1000}$
$\frac{42}{100}$	$\frac{194}{1000}$	$\frac{796}{1000}$	$\frac{304}{1000}$	$\frac{37}{1000}$	$\frac{7}{1000}$

DEFINITION.

DECIMAL FRACTIONS, or *Decimals*, are fractions written like whole numbers, and have a dot prefixed to them, to designate the value of their denominator.

The denominator of a decimal is always either 10 or some multiple of 10; and it is from the Latin word *decem*, which signifies *ten*, that the fraction derives its name.

The following table will show the numeration of decimals.

4.	7	6	5	2	8	4
Units	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths

The dot before the decimals is sometimes called **the *separatrix***, because it separates them from the integers.

RULES.

To write a decimal fraction, set down the numerator and put a dot before it; taking care to make as many figures in the numerator as there are ciphers in the given denominator. If there are not figures enough in the numerator, *prefix* ciphers to it, before the dot is put before it.

To read a decimal fraction, call the number, and then give it a denominator that is made up of 1 and as many ciphers as there are figures in the fraction.

To change a decimal fraction to a greater denominator, see first how many ciphers would be annexed to the given denominator, and then *annex* as many ciphers to the decimal.

When there are ciphers at the right of a decimal, the decimal may be reduced to a smaller denominator by erasing the ciphers.

Examples.

Reduce the following numbers to millionths.

1. .66.	4. .378.	7. .07654000.
2. .725.	5. .0042.	8. .0002480.
3. .04.	6. .07641.	9. .000075000.

ADDITION AND SUBTRACTION OF DECIMALS.

Rule.

Write tenths under tenths, hundredths under hundredths, &c., and proceed as in whole numbers.

APPLICATION.

1. 47.1235 + 6.135 + 7.98 + 4.36753 = 65.60603.

2. 7.1532 + 41.6 + 37.13798 + 2.645321 = 88.536501.

3. .4767 + .897245 + .4893 + .21769875 = 2.08094375.

4. .37 + 14 + 6.7 + 53.8 + 119.6743 + 160 = 354.5443.

5. .36 + 63 + 7.9 + .4754321 + 1234567.8 = 1234639.5354321.

6. 4 + .5 + .0009 + .000009 + 90000 + 4.0053 = 90008.506209.

7. 94 + .49 + .06 + .8907 + .0987 + .005 + .08 = 95.6244.

8. 43.001 + 100 + .4 + .87 + .53217 + .000943 = 144.804113.

9. 27.84 — 3.4567 = 24.3833.
10. .0765432 — .007654321 = .068888879.
11. 11 — .11 = 10.89.
12. 4321 — .13245 = 4320.86755.
13. 99 — .99 = 98.01.
14. 10000 — .000001 = 9999.999999.
15. 5000.0005 — 4000.00044 = 1000.00006.
16. 472.08 — 357.0005 + 27 — 136.3 = 5.7795.
17. 8.94367 — 6.532787 + 6.00005 = 8.410933
18. .00009 — .000008 — .0000007 = .0000813.
19. .004 — .0008 — .00007 + 6 = 6.00313.
20. 75 — .576 — .8972 — 4.843 = 68.6838.
21. 67 — 66.00009 = .99991.
22. 43 — 42.999999 = .000001.

MULTIPLICATION OF DECIMALS.

INDUCTIVE EXERCISES FOR THE SLATE.

1. How much is 5 times 4.14?

Ans. The method in which this would be done as vulgar fractions, is as follows. $4\frac{14}{100} = \frac{414}{100}$. And $\frac{414}{100} \times 5 = \frac{2070}{100}$ $20\frac{70}{100}$. Hence it seems, we may multiply the entire quantity without paying any regard to the separatrix, if we take care to point off as many decimals in the answer as there are in the multiplicand.

4.14
5
20.70

2. Here, .43 multiplied by 5, equals 215 hundredths, which = 2.15. The 2 is carried to the units.

8765.43
5
43827.15

3. Multiply 12.5 by 6. Ans. 75.0 or 75.
4. Multiply 1.67 by 9. Ans. 15.03.
5. Multiply 8.95 by 11. Ans. 98.45.
6. Multiply 537.467 by 367. Ans. 197250.389.
7. Multiply 476.895 by 27. Ans. 12876.165.
8. Multiply 17.9832 by 71. Ans. 1276.8072.

9. Multiply 876543 by .5.

We know the product will be the same as 876543 times $\frac{5}{10}$. Therefore, the product will have one decimal figure in it. Hence, when you multiply a whole number by a decimal fraction, multiply as with a whole number, and point off as many decimals as are in the multiplier.

876543
.5
438271.5

10. Multiply 37864 by 23.5. Ans. 889804.
11. Multiply 12 by 5.875. Ans. 70.5.
12. Multiply 32 by .04. Ans. 1.28.
13. Multiply 96 by .125. Ans. 12.
14. Multiply 596875 by 1.44. Ans. 859500.
15. Multiply 37864 by 2.09. Ans. 79135.76.
16. Multiply .3 by .5. Ans. $\frac{3}{10} \times \frac{5}{10} = \frac{15}{100}$.

In this example, we multiply the numerators together for the answer. And the denominator we find to contain as many ciphers, as the denominators of both the multiplicand and the multiplier. From this, we learn that there must be as many *decimal figures* in the product as in both the multiplicand and the multiplier.

17. Multiply 87654.3 by .5.

Here $\frac{5}{10}$ of $\frac{3}{10} = \frac{15}{100} = \frac{1}{10} + \frac{5}{100}$. Therefore, 5 hundredths is the first figure on the right. $\frac{5}{10}$ of $4 = \frac{20}{10}$, and the $\frac{1}{10}$ from the $\frac{15}{100}$ put with it, makes $\frac{21}{10} = 2.1$. Therefore 1 tenth is the next figure.

87654.3
.5
43827.15

As it will always happen that the number of decimals in the answer, will equal the number of decimals in the multiplicand and the multiplier put together; the sums may be performed as though they were whole numbers, and the proper number of decimals pointed off afterwards.

[18th sum.]	[19.]	[20.]
675.489	967.584	.27642
14.5	.236	.0345
3377445	5805504	138210
2701956	2902752	110568
675489	1935168	82926
9794.5905	228.349824	.009536490

RULE.

First, Place the multiplier under the multiplicand, putting the right hand figure of the multiplier under the right hand figure of the multiplicand, (without regard to the separatrix,) and then multiply as in whole numbers.

Second, Point off in the product, as many figures for decimals as there are decimals in the multiplicand and the multiplier counted together.—If there are not as many figures in the product as there are decimals in the multiplicand and multiplier, there must be ciphers prefixed to the product until there be as many.

APPLICATION OF THE RULE.

1. Multiply 25.238 by 12.17. Ans. 307.14646.
2. Multiply 4602 by .075. Ans. 345.15.
3. Multiply .234 by .061. Ans. .014274.
4. Multiply 9.604 by .19. Ans. 1.82476.
5. Multiply 4.67 by .276. Ans. 1.28892.
6. .486547 × .348. Ans. .169318356.
7. 89.6572 × .071. Ans. 6.3656612.
8. 4.367418 × .00043 = .00187798974.

9. $1.231 \times .276431 = .340286561$.
10. $.2 \times 4.76 = .952$.
11. $6.7 \times .002 = .0134$.
12. $.004 \times .006 = .000024$.
13. $.76 \times .492 = .37392$.
14. $.0003 \times .00007 = .000000021$.
15. $.0000006 \times 6000000 = 3.6$.
16. $42.89 \times .06 = 2.5734$.
17. $.2849 \times .4278 = .12188022$.

DIVISION OF DECIMALS.

INDUCTIVE EXERCISES FOR THE SLATE.

1. If 5 yds. of cloth cost $35.75, what is the cost of 1 yard?

Ans. 35.75 may be written $35\frac{75}{100}$, which is equal to $\frac{3575}{100}$. $\frac{1}{5}$ of $\frac{3575}{100} = \frac{715}{100}$, which $= 7.15$. Hence it appears that we may divide a decimal as we divide a whole number, if we take care to put a dot in the answer the moment we begin to use a decimal in the dividend.

$$5)35.75$$
$$7.15$$

The pupil may now perform the following sums, always taking care to put a dot in the quotient immediately before he uses the first decimal figure in the dividend.

2. What is one eighth of 1851.320? Ans. 231.415.
3. Divide 98.45 by 11. Ans. 8.95.
4. Divide 807.599 by 13. Ans. 62.123.

5. Divide 1.624 by 14. Ans. .116.

6. Divide 8,514 by 22. Ans. .387.

7. Divide 50.5845 by 135. Ans. .3747.

8. Divide .28092 by 12.

Ans. $\frac{28092}{100000} \div$ by $12 = \frac{2341}{100000}$ which $= .02341$.

This method of performing the sum shows that there must be as many figures in the answer as there are ciphers in the denominator. And this will not be the case unless we put a figure in the answer for every decimal figure in the dividend. Therefore in sums of this kind, the moment you come to the dot in the dividend, put a dot in the quotient, and set down after it, how many times the divisor is contained by using the first decimal figure, even if the answer is 0.

$$12)\overline{.28092}\ .02341$$

9. Divide .144 by 4. Ans. .036.

10. Divide .2996 by 7. Ans. .0428.

11. Divide .0648 by 12. Ans. .0054.

12. Divide .1248 by 52. Ans. .0024.

13. Divide .2176 by 16. Ans. .0136.

14. Divide .979776 by 144. Ans. .006804.

15. Divide 27 by 6.

Ans. 6 is contained in 27, 4 times and 3 remains. But 3 is equal to $\frac{30}{10}$; and 1 sixth of $\frac{30}{10} = \frac{5}{10}$. Hence, in decimals we shall only have to annex a cipher to the remainder, and then call the number tenths, and divide as before.

$$6)\overline{27}\ 4.5$$

16. Divide 6543.2 by 5.

Ans. $6543.2 \div 5 = 1308.6$ and $\frac{2}{10}$ over. But $\frac{2}{10} = \frac{20}{100}$; and $\frac{20}{100} \div 5 = \frac{4}{100}$. Therefore 4 will be in the hundredths place in the answer.

$$5)\overline{6543.2}\ 1308.64$$

7. Divide 476.72 by 37.

$$
\begin{array}{r|l}
37 & 476.72(12.88432+ \\
\hline
106 & \\
74 & \\
\hline
327 & \\
296 & \\
\hline
312 & \\
296 & \\
\hline
160 & \\
148 & \\
\hline
120 & \\
111 & \\
\hline
90 & \\
74 & \\
\hline
\end{array}
$$

In this example, after we have divided 476.72, we find $\frac{16}{100}$ remainder. This, $= \frac{160}{1000}$, which, divided by 37, $= \frac{4}{1000}$, and $\frac{12}{1000}$ remainder. This $\frac{12}{1000} = \frac{120}{10000}$, which, divided by 37, $= \frac{3}{10000}$ and $\frac{9}{10000}$ remainder. This $\frac{9}{10000} = \frac{90}{100000}$, which, divided by 37, $= \frac{2}{100000}$, and there is still a remainder.

In this sum, there would be a remainder, if the division were continued for ever. But by going as far as we have, we have obtained the true answer within $\frac{1}{100000}$ of 1. As such an answer is near enough for all practical purposes, we will consider the sum as performed. And so may our pupils, in the following sums, when they have have obtained five decimals.

18. Divide 222 by 365. Ans. .60821 +

19. Divide .6 by 94. Ans. .00638 +

20. Divide .1606 by 44. Ans. .00365.

21. Divide 48 by .3.

Ans. $48 = \frac{480}{10}$, which contains $\frac{3}{10}$ 160 times. Therefore when the divisor is a decimal, we must bring both dividend and divisor to the same denominator, and then divide as in whole numbers. In such cases, there will be no decimals in the quotient, unless more ciphers are annexed afterwards.

$$
\begin{array}{r|l}
.3 & 48.0 \\
\hline
 & 160 \\
 & \\
\end{array}
$$

In the following sums, when there is not the same number of decimals in the divisor as in the dividend, ciphers must be annexed to the least, until there is as many.

22. Divide 543.78 by 1.2. Ans. *The operation is the same as* 543.78 ÷ 1.20 = 453.15.

23. Divide 6.35 by .425. Ans. 14.941 +
24. Divide 7.735 by 3.25. Ans. 2.38

RULE.

If the divisor contains *no* decimals, divide as in whole numbers; taking care to put a separatrix in the quotient when you arrive at the separatrix in the dividend.

If there be a remainder after all the figures in the dividend have been used, the division may be carried farther by annexing ciphers.

If there are decimals in the divisor, first make the divisor and the dividend of the same denominator, and then divide as in whole numbers, without putting a separatrix in the quotient.

But if there are to be ciphers annexed to the dividend after the divisor and the dividend have been brought to the same denominator, a separatrix must be put in the quotient before that is done.

APPLICATION OF THE RULE.

1. Divide 4378.2 by .676. Ans. 6476.6272 +
2. Divide .826 by 48.1. Ans. .01717 +
3. Divide .736 by 431. Ans. .0017 +
4. Divide .134 by 1.87. Ans. .07165 +
5. Divide .896 by .548. Ans. 1.635 +
6. Divide .0002 by 37. Ans. .0000054 +
7. Divide .0026 by .003. Ans. .8666 +
8. Divide .01 by .00004. Ans. 250.
9. Divide .8 by 476.3. Ans. .001679 +
10. Divide 8 by 673. Ans. .011887072 +
11. Divide 496 by 37. Ans. 13.405405 +
12. At $3.27 a yd., how many yds. may be bought for $896.375? Ans. 274.12 + yds.

13. At $.97 a bushel, how many bushels of grain may be bought for $8.76? Ans. 9.0309 bush.

14. If 36.34 bushels of corn grow on one acre, how many acres will produce 674 bushels? Ans. 18.547 acres.

15. At $2.875 a gallon, how many gallons of wine may be bought for $48? Ans. 16.69565 gals.

16. At $3.666 a barrel, how many barrels of cider may be bought for $39? Ans. 10.63829 bbls.

17. If 1 yd. of cloth costs $5.5625, how much will ¼ of a yard cost? Ans. $1.3906.

18. If 4 yds. of cloth cost $18.125, what is the cost of 1 yd.? $4.53125.

As a vulgar fraction is the expression of a division, the numerator being the dividend, and the denominator a divisor; it is evident we may *perform* the division by the help of decimals. Hence the

RULE

For changing Vulgar Fractions to Decimals.

Annex ciphers to the numerator and divide by the denominator.

APPLICATION.

1. Change ¾ to a decimal. Ans. 3.00 ÷ 4 = .75.

¾ = ¼ *of* 3, *which* = ¼ *of* 30/10, *&c.* | 4)3 / .75

2. Change ⅕ to a decimal. Ans. .2.
3. Change ⅝ to a decimal. Ans. .625.
4. Change 3/15 to a decimal. Ans. .2.
5. Change ⅗ to a decimal. Ans. .6.
6. Change ⅞ to a decimal. Ans. .875.
7. Change 7/16 to a decimal. Ans. .4375.
8. Change 5/16 to a decimal. Ans. .3125.

Note.—There must always be one figure in the quotient for every cipher after the separatrix in the dividend.

9. Change $\frac{1}{125}$ to a decimal. Ans. .008.

10. Change $\frac{3}{192}$ to a decimal. Ans. .015625.

11. Change $\frac{14}{2560}$ to a decimal. Ans. .0546875.

Note.—2. When any remainder recurs the second time, it is a sign that the previous figure or figures in the quotient will continue to be repeated forever. Thus, $\frac{1}{3} = .3333333+$. Such decimals are called *repeating decimals*; and the figure that is repeated is called a *repetend*.

3. If the recurring figures embrace *two* or *more*, they are called *circulates*; and the decimal is a *circulating decimal*.

In such cases, the answer may be extended to as many figures as is thought necessary. But it is generally sufficient to write the repetend or circulate only once; taking care to place a point over the repetend, and over the *first* and last figures of the circulate.

12. Reduce $\frac{1}{6}$ to a decimal. Ans. .1666, or $.1\dot{6}$.

13. Reduce $\frac{6}{11}$ to a decimal. Ans. .545454, or $.\dot{5}\dot{4}$.

14. Reduce $\frac{1}{9}$ to a decimal. Ans. .1111, or $.\dot{1}$.

15. Change $\frac{11}{12}$ to a decimal. Ans. .9166, or $.91\dot{6}$.

16. Change $\frac{6}{7}$ to a decimal. Ans. .8571428, or $.\dot{8}57142\dot{8}$.

17. Change $\frac{13}{37}$ to a decimal. Ans. .351351, or $.\dot{3}5\dot{1}$.

18. Change $\frac{5}{6}$ to a decimal. Ans. .833, or $.8\dot{3}$.

Note.—4. When the division does not end in a repetend or circulate, the decimal is called an *approximate* decimal.

19. Reduce $\frac{5}{26}$ to a decimal. Ans. .1923+.

20. Reduce $\frac{11}{60000}$ to a decimal. Ans. .000183+.

21 Reduce $\frac{1}{23}$ to a decimal. Ans. .04347+

22. Reduce $\frac{9}{1428}$ to a decimal. Ans. .00631+.

23. Reduce $\frac{11}{14}$ to a decimal. Ans. .78571+

24. Reduce $\frac{10}{13}$ to a decimal. Ans. .76923+.

To change Decimals to Vulgar Fractions.

RULE.

Take the decimal without its point, for a numerator; and under it, write the decimal denominator. Then reduce it to its lowest terms.

SUMS FOR EXERCISE.

1. Reduce .625 to a vulgar fraction.

Ans. $\frac{625}{1000}$, *which equals* $\frac{5}{8}$. | $\frac{625}{1000} \div 5 = \frac{125}{200} \div 5 = \frac{25}{40} \div 5 = \frac{5}{8}$.

2. Change .4375 to a vulgar fraction. Ans. $\frac{7}{16}$.

3. Change .3125 to a vulgar fraction. Ans. $\frac{5}{16}$.

4. Change .015625 to a vulgar fraction. Ans. $\frac{1}{64}$.

5. Change .0546875 to a vulgar fraction. Ans. $\frac{7}{128}$.

Note 1.—If the decimal is a repetend or circulate, instead of using 1 with ciphers, we use as many 9's as there are figures in the repetend or circulate.

6. Change .$\dot{1}0\dot{5}$ to a vulgar fraction. Ans. $\frac{105}{999} = \frac{35}{333}$.

7. Change .$\dot{4}2\dot{3}$ to a vulgar fraction. Ans. $\frac{47}{111}$.

8. Change .000$\dot{1}$ to a vulgar fraction. Ans. $\frac{1}{9999}$.

9. Change .$\dot{4}76\dot{1}$ to a vulgar fraction. Ans. $\frac{4761}{9999}$.

10. Change .$\dot{3}56\dot{4}$ to a vulgar fraction. Ans. $\frac{356}{1001}$.

Note 2.—If the decimal is a mixed circulate, (that is, if it has figures on the *left* of the recurring figures;) from the whole decimal, subtract the left hand figures for the numerator; and for the denominator, annex as many ciphers as there are left hand figures, to as many 9's as there are figures in the circulate.

11. Change .35$\dot{2}\dot{7}$ to a vulgar fraction.

Ans. 3527 — 35 = 3492. $\frac{3492}{9900} = \frac{97}{275}$.

12. Change .5$\dot{8}\dot{1}$ to a vulgar fraction. Ans. $\frac{23}{99}$.

13. Change .8$\dot{3}$ to a vulgar fraction. Ans. $\frac{5}{6}$.

To reduce a Denominate Fraction to Decimals of a higher Denomination.

RULE.—Bring the given quantity to a vulgar fraction of the required integer; and then change that vulgar fraction to a decimal.

SUMS FOR EXERCISE

1. 7 hours are $\frac{7}{24}$ of a day. What decimal is equal to $\frac{7}{24}$, and what decimal of a day is 7 hours? Ans. .2916+

2. What decimal of a day equals 10 hours?
Ans. .4166 + day.

3. What decimal of an hour equals 44 minutes?
Ans. .733 + hour.

4. What decimal of a foot equals 9 in.? Ans. .75 foot.

5. What decimal of a £. equals 15 shillings? Ans. .75£.

6. What decimal of a shilling equals 8d.? Ans. .66+s.

7. What decimal of a penny equals 1qr.? Ans. .25d.

8. What decimal of a lb. equals 14 oz.? Ans. .875 lb.

9. What decimal of a qr. of cwt. equals 25 lbs.?
Ans. .892857 cwt.

10. What decimal of a cwt. equals 72 lbs.?
Ans. 64285714 + cwt.

11. What part of a C. equals 58 lbs.? Ans. .58C.

12. What part of a gallon is 3 qts. 1 pt.?—and what decimal equals it?
Ans. 3 qts. 1 pt. = $\frac{7}{8}$ of a gallon; = .875 gal.

13. What part of a yd. is 3 qrs. 1 n.;—and what decimal equals it? Ans. .8125 yd.

14. What decimal of a yd. is equal to 2 qrs. 3 n.?
Ans. .6875 yd.

15. What decimal of a £. equals 10s. 9d.?
Ans. .5375£

16. What decimal of a day equals 10 h. 30 m. 15 s.?
Ans. 43767 + day

RULE 2.—It is sometimes more convenient to begin with the lowest denomination, and divide as in Reduction of Denominate Fractions. In this method, care must be taken to prefix to each quotient the given integer of that denomination.

17. What decimal of a £. equals 19s. 4½d.?

Ans. ½d. = .5 *of a penny. And there are as many shillings as there are* 12d. *contained in* 4.5d., *which is* .375s. *And there are as many £. as there are collections of* 20s. *contained in* 19.375s., *which is* .96875£.

2)1
12)4.5d.
20)19.375s.
.96875£.

☞ The pupil may perform the following by both rules.

18. Reduce 3 pks. 4 qts. to the decimal of a bushel.
Ans. .875 bush.

19. Reduce 13 acres 1 rood 14 rods to a decimal of an acre.
Ans. 13.3375 acres.

20. Reduce 17 dwt. 13 grains to the decimal of an ounce.
Ans. .877 oz.

21. Reduce 17s. 6¾d. to decimals of a £.
Ans. .878125£.

22. Reduce 6 oz. 15 dwt. 18 grs. to decimals of a pound Troy.
Ans. .565625 lb.

23. Reduce 3 pks. 6 qts. 1 pt. to decimals of a bushel.
Ans. .953125 bush.

24. What decimal of a cwt. equals 3 qrs. 21 lbs. 10 oz.?
Ans. .94308 cwt.

25. What decimal of a yd. equals 1 ft. 11 in.?
Ans. .6388+ yd.

26. What decimal of a yd. equals 1 ft. 11.75 in.?
Ans. .65972+ yd.

27. What decimal of a yd. equals 1 qr. 3.47 n.?
Ans. .466875 yd.

28. What decimal of a bushel equals 3 pks. 5 qts. 1.734 pts.?
Ans. .93334 + bush.

29. What decimal of a cwt. equals 3 qrs. 18 lbs. 11.437 oz.? Ans. .917 + cwt.

30. What decimal of a lb. equals 4 dr. 2 scr. 15 gr.? Ans. .0512152 + lb.

31. What decimal of an acre equals 125 rods? Ans. .78125 acre.

32. What decimal of a cord equals 100 ft. 1625 in.? Ans. .788596 + cord.

33. What decimal of a degree equals 36′ 40.375″? Ans. .611215° +

34. What decimal of a £. equals 14s. 7.458d.? Ans. .731075£.

To reduce a Decimal Fraction to its proper value.

RULE.

REDUCE the given decimal to the next lowest denomination as in whole numbers; and then point off as many decimal places as there are in the given decimal.

Reduce the *remaining decimal* to the next lowest denomination in the same manner; and so on as far as necessary.

APPLICATION.

What is the value of .625£.?

Ans. $\frac{625}{1000}$ of a £ = $\frac{625}{1000}$ of 20s. which is $\frac{12500}{1000}$ of a shilling; or 12s. and $\frac{500}{1000}$ of a shilling. $\frac{500}{1000}$ of a shilling = $\frac{500}{1000}$ of 12d., which is $\frac{6000}{1000}$ of a penny; or 6d. Therefore, the answer is 12s. 6d.

.625£
20
12.500s.
12
6.000d.
Ans. 12s. 6d.

2. What is the value of .478£.? Ans. 9s. 6½d.

3. What is the value of .5625 of a yd.? Ans. 2 qrs. 1 nail.

4. What is the value of .736 of a gal.? Ans. 2 qts. 1 pt. 3 gls.

5. What is the value of .37 of a day?
Ans. 8 h. 52 m. 48 s.

6 What is the value of .8125 of a bush.?
Ans. 3 pks. 2 qts.

7. What is the value of .785 of a £.? Ans. 15 s. 8.4 d.

8. What is the value of .86 of a cwt.?
Ans. 3 qrs. 12 lbs. 5.12 oz.

9. What is the value of .761 of a day?
Ans. 18 h. 15 m. 50.4 s.

10. What is the value of .3375 of an acre?
Ans. 54 rods.

11. What is the value of .025 of an acre? Ans. 4 rods.

12. What is the value of .125 of a gal.? Ans. 1 pint.

13. What is the value of .0625 of a shilling? Ans. $\frac{3}{4}$ d.

14. What is the value of .625 of a cwt.?
Ans. 2 qrs. 14 lbs.

15. What is the value of .0375 of a £.? Ans. 9 d.

16. What is the value of .7824 of a foot?
Ans. 9.3888 in.

17. What is the value of .6548 of a foot?
Ans. 7.8576 in.

MISCELLANEOUS EXERCISES IN DECIMALS.

1. WHAT is the cost of 1 yd. 3 qrs. 2 n. of broadcloth, at $6.375 per yd.?

Ans. 1 yd. 3 qrs. 2 n. = $1\frac{14}{16}$ yd., which = 1.875 yd.
And 1.875 times $6.375 = $11.953 +.

2. What is the cost of 3 gals. 1 qt. of wine, at $2 per gallon?
Ans. $6.50.

3. What cost 3 cwt. 1 qr. 18 lbs. of sugar, at 8 dollars a cwt.? Ans. $27.285 +

4. What cost 4 cwt. 1 qr. 10 lbs. of iron, at 5 dollars a cwt.? Ans. $21.695 +

5. What cost 18 cwt. 17 lbs. of rice, at 3 dollars a cwt.? Ans. $54.455 +

6. What cost 27 gals. 3 qts. 1 pt. of wine, at $1.375 a gallon? Ans. $38.328 +

7. What cost 17 lbs. 11 oz. of sugar, at $.10 a lb.? Ans. $1.768 +

8. What cost 18 yds. 1 qr. 3 n. of calico, at $.125 a yd.? Ans. $2.304 +

9. What cost 4 yds. 3 qrs. 1 n. of cloth, at 1£. 12 s. 9 d. a yd.? Ans. 7£. 17 s. 6¾ d.

10. What cost 10 yds. 1 qr. 1 n. of cloth, at 2£. 9 s. 7½ d a yd.? Ans. 25£. 11 s. 8 d.

11. What is the cost of 27.376 lbs. of tea, at $.625 a lb.? Ans. $17.11.

12. What will 750 peaches come to, at $3 a hundred? Ans. $22.50.

13. What will 5672 feet of boards come to, at $14.25 a thousand? Ans. $80.826.

14. What will 337 feet of boards come to, at $20 a thousand? Ans. $6.74.

15. What is the cost of 7465 feet of plank, at $39 a thousand? Ans. $291.13½.

16. What is the value of 3761 feet of joists, at $9.37½ a thousand? Ans. $35.259 +.

NOTE.—This is a convenient rule for measuring surfaces and solids.

Rule.—To find the contents of a surface, multiply its length by its breadth.

17. How many square inches in a board 6 inches long and 5 inches broad? Ans. 30 sq. in.

18. How many square feet in a board 18 feet long and 2 feet wide? Ans. 36 sq. ft.

19. How many square feet in a board 16½ feet long and 1¼ of a foot wide? Ans. 28.875 ft.

20. How many square feet in a board 16 ft. 6 in. long and 1 ft. 6 in. wide? Ans. 24¾ sq. ft.

21. How many square feet in a board 20 ft. 7 in. long and 2 ft. 3 in. wide? Ans. 46.312425 ft.

22. How many feet in a board 17 ft. 11 in. long and 3 ft. 2 in. wide? Ans. 56.7347 sq. ft.

23. How many feet in a board 15 ft. 9 in. long and 2 ft. 7 in. wide? Ans. 40.68225 ft.

24. How many feet in a board 15 ft. 8 in. long and 2 ft. 5 in. wide? Ans. 37.849 sq. ft.

25. How many feet in a board measuring 14 ft. 10 in. by 2 ft. 3 in.? Ans. 33.37425 ft.

26. How many feet in a plank measuring 24 ft. 7 in. by 11 in.? Ans. 22.532777 ft.

27. How many feet in a board 18 ft. 1 in. by 1 ft. 4 in.? Ans. 24.044 ft.

28. How many feet in a room 16 ft. 3 in. by 18 ft. 7 in.? Ans. 301.97175 ft.

29. How many feet in a floor 14 ft. 2 in. by 17 ft. 3 in.? Ans. 244.3635.

30. How many square inches in a pane of glass 7 inches by 9 inches? Ans. 63 inches.

31. How many square inches in a pane of glass 12 by 14? Ans. 168.

32. How many feet of boards will it require to lay a floor 16 ft. long and 14 ft. 10 in. wide? Ans. 237.328.

33. How many square feet in a piece of land 137 feet long and 68 feet 5 inches wide? Ans. 9372.992 ft.

34. How many square rods in a piece of land 37 rods long and 33 rods 10 ft. wide? Ans. 1243.422 rods.

35. How many acres in a piece of land 48 rods 8 feet long and 35 rods 11 feet wide?
Ans. 10 acres, 129.28796768 rods.

36. How wide must be a piece of land that is 18 rods long to be an acre?
Ans. 8.8888 rods, or 8 rods 14 ft. 7.998 in.

37. How many square feet in a yard of carpeting that is 2 ft. 7 in. wide? Ans. 7.75 sq. ft.

38. How many yards of carpet, 2 ft. 7 in. wide, will it take to cover a floor 14 ft. by 16 ft. 6 in.?
Ans. 29.8 yards.

39. The wall on one side of a room is 16 ft. 4 in. long and 10 ft. 5 in. high? What is its square surface?
Ans. 170.1245 sq. ft.

40. How many yards of paper, just 2 feet wide, will it take to cover a wall 10 ft. 5 in. high, and 16 ft. 4 in. long?
Ans. 28.356 yards.

41. How many yards of carpet, 2 feet 9 inches wide, will cover a floor 14 feet 7 inches long and 11 feet 10 inches wide? Ans. 20.916 yds.

42. How much paper will it take to paper a room measuring as follows: One side 14 ft. long, 10 ft. 5 in. high, with two windows in it, each measuring 3 square yards; another side of the same size, without any windows; another side 12 ft. 6 in. long 10 ft. 5 in. high, with a fireplace in it measuring 8 square yards; and another side of the same size, with a door in it 4 ft. by 8 ft.?
Ans. 394.048 sq. ft.

Explanation.—A solid is a figure that has length, breadth, and thickness.

Rule.—To find the contents of a solid, multiply the length by the breadth, and that product by the thickness.

43. How many solid feet in a square stick of timber 21 feet long, 3 feet 7 inches wide, and 2 feet 3 inches thick?
Ans. 169.29675 ft.

44. How many solid feet in a stick of timber 18 ft. 2 in. long, 3 ft. 3 in. wide, and 2 ft. 11 in. thick?
Ans. 172.159 ft.

45. How many solid feet in a stick of timber 15 ft. 8 in. long, 1 ft. 8 in. wide, and 1 ft. 5 in. thick?
Ans. 36.95697 ft.

46. How many solid feet in a stick of timber 27 ft. long, that is 1 ft. 7 in. by 2 ft. 3 in.? Ans. 96.185475 ft.

47. How many solid feet in a stick of timber, 24 ft. 6 in. long and 2 ft. 5 in. through, both ways? Ans. 143.0789 ft.

48. How many tons of hewn timber are there in 3 sticks, each 21 ft. 3 in. long, and 17 in. wide, and 14 in. thick?
Ans. 2.105 tons.

49. How many tons of timber in 4 sticks, each 18 feet long and 14 by 18 in. through? Ans. 2.5185 tons.

50. How much wood in a load, measuring 8 feet long, 4 ft. wide, and 5 ft. 3 in. high? Ans. $1\frac{40}{128}$ cords.

51. How many feet of wood, in a load 8 feet long, 3 feet 10 in. wide, and 3 ft. 9 in. high? Ans. 114.99 ft.

52. How many feet of wood in a load 7 ft. 8 in. long, 3 ft. 7 in. wide, and 3 ft. 3 in. high? Ans. 89.2686 ft.

As in the foregoing questions, the most easy method is to bring the lower denominations to decimals of the higher, it would be well for those who measure timber, boards, &c., to divide their rules into tenths of a foot, and those tenths into tenths of tenths, or hundredths of a foot.

There is also another method of performing questions of this kind, called *duodecimals*.

PER CENTAGE.

INDUCTIVE EXERCISES FOR THE SLATE.

1. A son wished to borrow $100 of his father, and keep it a year. His father said that he would lend it to him, if he would pay him 6 dollars for the use of it. How much would the son owe the father at the end of the year?

Ans. He would owe $100 borrowed money, and $6 for the use of it.

2. A gentleman wished to borrow of a miser 100 dollars for a year. The miser said he would lend it to him, if he would pay him at the end of the year, 25 dollars for the use of the $100. How much would the gentleman owe the miser at the end of the year?

3. Why would he owe him 125 dollars?

4. A gentleman borrowed of a friend 100 dollars; and at the end of the year paid it with 7 dollars more. Why did he pay him that 7 dollars?

Money that is paid for the use of borrowed money is called *interest*. The money that is lent is called principal, and the principal and interest added together is called the *amount*. The use of money is generally supposed to be worth at the rate of 6 cents for every dollar, and therefore most of the states have made a law that it should be no more. In New York 7 cents for a dollar is allowed.

5. What part of a dollar is 6 cents?

The proportion of one number to another is sometimes called the *rate*. So we say we pay interest at the rate of 6 cents for a dollar. If we give 6 cents for a hundred cents, at the same rate we would give six dollars for a hundred dollars, and so in general 6 of any thing for a hundred of the same.

This is generally expressed by the use of the Latin word *per*, which means *by*, and *cent.*, which means a *hundred.* Therefore when we say 6 *per cent.*, we mean 6 *for a hundred.*

6. Thomas wished to borrow 2 dollars of his brother James; and James agreed to let him have it for 6 per cent. interest. What would be the interest of the 2 dollars at the end of the year?

If Thomas paid James 6 cents for the use of 100 cents, he paid $\frac{6}{100}$ as much as 100 cents. At the same rate for any sum of money he would pay $\frac{6}{100}$ as much as that sum. Therefore in this case he would pay $\frac{6}{100}$ of $2.

The easiest method of finding how much this is, is to multiply the principal by .06, as in the margin. The answer is $.12; or, 12 cts.

2
.06
.12

7. What would be the interest of the 2 dollars at the end of the year at 5 per cent.?

Ans. Five per cent. is .05; and $\frac{5}{100}$ of $2 is $.10; or 10 cts.

2
.05
.10

8. What is the interest of 4 dollars for a year at 6 per cent.? Of 8 dollars? Of 9 dollars? Of 12 dollars?

9. What is the interest of 4 dollars for a year at 6 per cent.? Of the same for 2 years? For 6 years?

10. At 6 per cent., what is the interest of 50 cents for one year? Of 1 dollar 50 cents? Of $3.50? Of $6.50?

11. A merchant living in New York, desired a merchant in Philadelphia to sell some goods for him. The Philadelphian agreed to do it for 5 per cent. What did the New Yorker give the Philadelphian for selling 300 dollars worth? What for selling 700 dollars worth?

Money that we pay to another for buying or selling goods for us, is called *commission.* There are some merchants who own no goods themselves, but buy and sell for others. They are called *commission merchants.*

12. A merchant, as he was sending out a vessel worth 2500 dollars, told another that he was afraid it would be lost. His friend said that if he would give him 4 per cent. of what it was worth, he would pay him the 2500 dollars if the vessel was lost. The merchant agreed to do so. What did he pay him?

Money that is paid in such cases is said to be paid for insurance, because the person to whom it was paid *insures either the vessel or its worth* to the owner. Business people call it the *premium* for insurance.

13. A bookseller brought some books from England. When he arrived, he found there was a law that he must pay a certain rate per cent. of their cost towards the support of government. How much must he pay, supposing the books to cost 200 dollars, and the rate to be 15 per cent.?

Money that is paid to government for the privilege of bringing goods from other countries, is called *duty*.

14. A company of gentlemen agreed to put some money together to build a bridge, that they might make money from the toll. They agreed to make 30 shares of it, and pay $100 for each share. Afterwards other people found that they made so much money on it, that they were willing to give $110 a share. What was the value of those shares per cent.? Ans. 110 per cent.

Money used in this way is called *stock*. In this example the stock in each share was $100.

As *per cent.* is written like decimals of hundreds, it is easy to change the vulgar fraction to per cent; thus, $\frac{1}{20}$ is equal to .05, which may be read 5 *per cent.* The only thing to be remembered is, that there must be two decimals, in order that they may be hundreds.

15. Change $\frac{1}{50}$ to per cent, or decimals. Ans. .02.

16. Change $\frac{1}{25}$ to per cent. Ans. .04.

17. Change $\frac{1}{10}$ to per cent. Ans. .10.
18. Change $\frac{1}{5}$ to per cent. Ans. .20.
19. Change $\frac{1}{4}$ to per cent. Ans. .25
20. Change $\frac{1}{2}$ to per cent. Ans. .50.

It is also to be remembered that there must be only *hundredths*. Thus, $\frac{1}{3}$ changed to per cent. is .33$\frac{1}{3}$, or 33$\frac{1}{3}$ per cent.

21. Change $\frac{1}{6}$ to per cent. Ans. .16$\frac{2}{3}$.
22. Change $\frac{2}{3}$ to per cent. Ans. .66$\frac{2}{3}$
23. Change $\frac{4}{9}$ to per cent. Ans. .44$\frac{4}{9}$.
24. Change $\frac{7}{15}$ to per cent. Ans. .46$\frac{2}{3}$.

25. What per cent. is 6 cents on a dollar? 14 cents on a dollar?

DEFINITION.

PER CENTAGE is a method of computing by means of a fraction whose denominator is 100.

The *rate per cent.* is the value of the decimal fraction and is always expressed by the numerator.

The rate per cent. that one number bears to another is found by the following

RULE.

First, Form a vulgar fraction, by making a numerator of the number you wish to compare, and a denominator of the number you wish to compare with.

Second, Change the vulgar fraction to a decimal of *two figures*, which will be the answer. If the reduction of the vulgar fraction to a decimal does not stop at hundredths, the remainder may be expressed as a fraction of per cent.

The fraction of per cent. may be either vulgar or decimal. Thus, 3$\frac{1}{2}$ per cent. may be written .03$\frac{1}{2}$, or .035. So, 4$\frac{3}{4}$ per cent. may be written .04$\frac{3}{4}$, or .0475.

APPLICATION OF THE RULE.

1. A MAN bought a hogshead of sugar for $96, and sold it so as to gain $24. What did he gain per cent.?

Ans. He gained $\frac{24}{96}$ of the first cost, and $\frac{24}{96} = .25$. Therefore, he gained 25 per cent.

$\frac{24}{96} = \frac{1}{4} = .25$

2. If a man gain 15 dollars on 75 dollars, what does he gain per cent.? Ans. 20 per cent.

3. If a man gain 20 dollars on 60 dollars, what does he gain per cent.? Ans. $33\frac{1}{3}$ per cent.

4. If a man gave 217 dollars for a quantity of cloth, and sells it for 246 dollars, what does he gain on the whole? And how much per cent. on the first cost?

Ans. $13\frac{70}{217}$ per cent.

5. If a man gave 376 dollars for a quantity of iron ware, and sold it for 408 dollars, what did he gain on the whole? And how much per cent. on the first cost?

Ans. $8\frac{24}{47}$ per cent.

6. If a man gave $1700 for a quantity of flour, and sold it for $2168, what did he gain per cent. on the first cost?

Ans. $27\frac{9}{17}$ per cent.

7. If a man gave $800 for a quantity of cotton yarn, and sold it so as to gain 353 dollars, what was his gain on the first cost? Ans. $44\frac{1}{8}$ per cent.

8. If a man gave $274 for a gig, and sold it for 255 dollars, what did he lose on the whole? And how much per cent. on the first cost? Ans. $6\frac{124}{137}$ per cent.

9. If a man gave $670 for a quantity of broadcloth, and sold it for $629.80, what did he lose on a dollar?

Ans. 6 per cent.

10. Bought cloth at 25 cents a yard, and sold it at 30 cts. What is the gain per cent.? Ans. 20 per cent.

11. Bought cloth at 31 cents a yard, and sold it for 28 cts. What is the loss per cent.? Ans. $9\frac{21}{31}$ per cent.

12. A merchant bought 81 gallons of molasses for $22, and sold it for 29 cents a gallon. What is his gain per ct.?
Ans. $6\frac{17}{22}$ per cent.

13. A merchant bought 12 C. of sugar for 96 dollars, and sold it for 14 cents a pound. What is his gain per ct.?
Ans. $37\frac{1}{2}$ per cent.

PER CENTAGE, AS APPLIED TO BUSINESS.

1. MANY business calculations are by per cent. The most important of these are *Profit and Loss*, *Commission*, *Bankruptcy*, *Stocks*, *Exchange*, *Insurance*, *Interest*, *Discount*, *Duties*, and *Taxes*.

In all these operations, the per centage is found by the following

RULE

Reduce the rate per cent. to a decimal form, and multiply the sum by it, as in decimal fractions.

APPLICATION.

PROFIT AND LOSS.

1. A MERCHANT sold a quantity of goods, that cost $150, so as to gain 20 per cent. What did he gain?
Ans. $30.

2. A merchant sold goods that cost $437, so as to gain 25 per cent. What did he gain? And what did he sell them for? Ans. Gain, $109.25; and sold for $546.25.

3. What will be the gain on $47.30, at the rate of 18 per cent.?
Ans. $8.51.

4. A merchant sold goods, that cost him $765, so as to lose 4 per cent. What was his loss? And what did he sell them for? Ans. Loss, $30.60; sold for $734.40.

5. A merchant sold goods, that cost $6574, so as to gain 37½ per cent. What did he gain? And what did he sell them for? Ans. Gain, $2465.25; sold for $9039.25.

6. A merchant bought tea for 90 cents per pound. What must he sell it for to gain 20 per cent.?
Ans. $1.08 per lb.

7. A merchant bought some wine, at $1.25 per gallon; but, as it was not so good as he expected, he is willing to lose 18 per cent. by it. At how much a gallon must he sell it? Ans. $1.02½.

8. A merchant having cambric, that cost $1.08 per yard, wishes to sell it so as to gain 15 per cent. At how much a yard must he sell it? Ans. $1.24.

9. A merchant wishes to gain 33⅓ per cent. on cloth that cost him $4.37½ per yard. At what rate must he sell it?
The multiplier is .33⅓. Ans. $5.83.

10. A merchant is willing to lose 12½ per cent. on sugar that cost him 8 cents per lb. At what rate must he sell it?
Ans. 7 cts. per lb.

11. A merchant bought a hogshead of molasses for $20.75, and wishes to receive a profit from it of 20 per cent. At how much a gallon must he sell it, if it gauges 72 gallons? Ans. 0.34\frac{7}{72}$.

COMMISSION.

1. An auctioneer sold a house for $6795.45. What is his commission at 2 per cent.? Ans. $135.91.

2. My agent has bought for me goods to the amount of $758.34. What is his commission at 3 per cent.?
Ans. $22.75.

3. My agent sold for me goods to the amount of $875.37½. What is his commission at 5 per cent.
Ans. $43.77.

4. My agent has paid for me debts amounting to $349.49; and sent his account to me, charging 2½ per cent. commission. What will be his commission, and what will be the whole amount? Ans. Com. $8.74; amount $358.23.

5. My friend in England paid for me $771; and has sent me his account, charging 4 per cent. commission. What is the whole amount of his account?
Ans. $801.84.

6. My agent has sent me a bill of goods bought for me, amounting to $1238, accompanied by a charge of 5½ per cent. commission. What is the whole amount?
Ans. $1306.09.

7. What is the commission on $1762.36, at 6¼ per cent.?
Ans. 110.14¾.

8. What is the commission on $49.14, at 3½ per cent.?
Ans. $1.72.

9. What is the commission on 425£. at 7½ per cent.
Ans. 30£. 16s. 3d.

10. What is the commission on 85£. at 4¾ per cent.?
Ans. 4£. 9d.

11. What is the commission on 176£. 4s. 10d. [*reduce it to decimals,*] at 4½ per cent.? Ans. 7£. 18s. 7¼d.

☞ 2s. = $\frac{1}{10}$ of a £. or .1£. Therefore 4s. = .2£. 10d. = 40 qrs. which = $\frac{40}{960}$£. Now as every $\frac{24}{960}$ = $\frac{25}{1000}$, we know that when above $\frac{12}{960}$, the number of $\frac{}{960}$ will be as many $\frac{}{1000}$, and a little more than half of another $\frac{}{1000}$. It will be near enough to call it one more. When we get to $\frac{24}{960}$, it is just 1 more in $\frac{}{1000}$. When we get up to $\frac{36}{960}$, it is 37½ thousandths; and when over $\frac{36}{960}$, it is almost 2 more in $\frac{}{1000}$. Therefore we will say $\frac{40}{960}$ = $\frac{42}{1000}$ or .042£. The whole decimal will be 176.242.

12. What is the commission on 94£. 10s. 6d. at 3¼ per cent.? Ans. 3£. 1s. 5¼d.

13. What is the commission on 126£. 18s. 4½d. at 10 per cent.? Ans. 12£. 13s. 9¾d.

INSURANCE.

1. A MERCHANT has sent a ship to sea, which, with her cargo, is worth $61765. By giving 2½ per cent. on the whole worth, he can get it all insured. How much must he give? Ans. $1544.12½.

2. An insurance company,* in Philadelphia, insured a ship's cargo, valued at $9640, for 4½ per cent. What is the amount of the premium? Ans. $433.80.

3. What is the premium for insuring a house against fire, if the house is valued $3750, at 1 per cent.?
Ans. $37.50.

4. What sum must be paid to insure a house valued at $6700, against loss or damage by fire, at 1¼ per cent.?
Ans. $83.75.

5. What is the amount of a premium of ¾ per cent. for insuring a house valued $25775? Ans. $193.31.

6. A gentleman had his house, valued at $18500, insured for a year, at ¾ per cent.; but the next year he was obliged to pay 1⅛ per cent. What was the amount of the premium the first year?—and how much greater was that of the next year?

Ans. The premium the first year was $138.75; that of the next year was $69.37½ more.

7. A merchant sent a vessel to sea, worth about $20000, for which he would have to pay a premium of 3½ per cent. He therefore thought that he would run some risk himself, and had the policy (*the instrument of writing in which the agreement was made*) made out for $15000. What is the amount of the premium to be paid? And how much did he save by reducing the policy?

Ans. He paid $525; and saved $175.

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* Insurance is generally made by companies incorporated for that purpose by government.

8. A merchant has goods in his store, for which he takes a policy of $25433 of a fire insurance company, at 1¾ per cent. premium. What does he pay? Ans. $445.07¾.

9. A China merchant had a vessel, valued in the policy at $50800, insured at 5½ per cent. What did the premium amount to? Ans. $2794.

10. A gentleman obtained insurance upon his house, valued at $5400, at ¾ per cent., and upon his furniture, valued, in policy, at $1575, for ¼ per cent. What was the amount of both premiums? Ans. $44.43¾.

11. What premium must be given for insuring a stock of goods, valued in policy at 4675 dollars, against fire, at 2 per cent. per year, supposing the policy runs for 6 months? Ans. $46.75.

12. What premium must be paid for insuring a house against fire, at 2½ per cent. a year, supposing the policy runs for $2050 for 9 months? Ans. $34.59⅜.

13. A gentleman, going to sea, thought that if he should be lost, his family would suffer. He therefore went to an insurance office, and wished to insure $1000 to his family, on condition of his death. What sum must he pay, supposing the premium demanded be 8 per cent.? Ans. $80.

14. What must be paid on a life insurance of $6000, supposing the premium to be 15 per cent.? Ans. $900.

BANKRUPTCY.

1. A MAN, failing in trade, owes as follows, viz. To A, $242.10; to B, $29.04; to C, $89.25; to F, $327; to G, $948.05; to H, $1127; and to K, $12.50. His property is valued at $1403.37½. How much per cent. can he pay: and how much can each creditor receive?

Ans. He pays 50½ per cent.; and A receives $122.26; B, $14.66½; C, $45.07; F, $165.13; G, $478.76½; H, $569.13½; K, 6.31¼.

3. A man, failing in trade, finds that he can pay but 34 per cent. on his debts. How much can he pay to L, whom he owes $729.50; to M, whom he owes $3500; to N, whom he owes $312.25; to O, whom he owes $1748.37½; and to P, whom he owes $98.10.

Ans. To L, $248.03; to M, $1190; to N, $106.16½; to O, $594.44½; and to P, $33.35.

3. A person owes the following sums: To Q, $24.12½; to R, $91.50; to S, $18.75; to T, $128.02; to U, $896.10; to V, $1142; to W, $41.62½; to X, $11.75; to Y, $50; and to Z, $423.84. He can pay only 75 per cent. How much can he pay on each debt; and how much will they all amount to, adding in the fractions of cents?

Ans. The whole amount is $2120.78.

STOCKS AND EXCHANGE.

When stock can be sold for more money than it originally cost, we say it is *above par*; and the difference is styled *advance*.

When it cannot be sold for as much as it cost, we say it is *below par*; and the difference is styled *discount*.

1. What is the value of $34000 of United States bank stock, at 112½ per cent.? Ans. $34000×1.12½=$38250.

2. What is the value of $650 of stock in a bridge, at 98½ per cent.? Ans. $640.25.

3. What is the value of $3720 in railroad stock, at 107 per cent.? Ans. $3980.40.

4. What is the value of $728 in canal stock, at 100¼ per cent.? Ans. $731.64.

5. What is the value of $896 in bank stock, supposing it ¼ per cent. below par? Ans. $891.52.

6. What is the value of $1400 in turnpike stock, supposing it to be ¼ per cent. above par? Ans. $1403.50.

7. What is the value of $3750 of stock in an insurance office, at 127 per cent.? Ans. $4762.50.

8. What is the value of $1700 of railroad stock, at 2 per cent. above par? Ans. $1734.00.

9. What is the value of $470 of steamboat stock, at 5½ per cent. below par? Ans. $444.15.

10. What is the value of $37 in bank bills, worth 99½ per cent.? Ans. $36.81½.

11. What is the value of $89 in bank bills, worth 98 per cent.? and what is the loss on them? Ans. $87.22.

12. What is the value of $370 in bank bills, worth 99¾ per cent.? and what must I lose to get them exchanged? Ans. Value, $369.07½.

13. What is the value of $456 in bank bills, worth 99¾ per cent.? and what is the loss in exchange? Ans. $454.86.

14. What is the value of $95 in bank bills, at 4½ per cent. below par? Ans. $90.72½.

15. What is the value of $107 in bank bills, 3½ per cent. below par? Ans. $103.25½.

16. A gentleman invested $875 in stock, which has since fallen 7¼ per cent. What is the worth of his share? Ans. $811.56¼.

17. A merchant bought 6 shares of stock for $875.75. The stock afterwards fell ⅞ per cent.? What was his loss? Ans. $7.66.

18. What is the value of $365 in bank notes, at ⅝ per cent. below par? Ans. $362.26¼.

19. What is the loss in the exchange of $578 in bank notes, at 1¼ per cent. below par? Ans. $7.22½.

20. What is the exchange of $67 in bank notes, at 2½ per cent. below par? Ans. $65.32¾.

21. What is the premium on $655 in bank bills, at ¼ per cent.? Ans. $1.63¾.

SIMPLE INTEREST.

1. What is the interest of $375, for one year, at 6 per cent.? Ans. $22.50.

2. What is the interest of $68.75, for 3 years, at 6 per cent. per annum?

Ans. *The rate for 3 years will be* .18; *therefore the interest will be* $\frac{18}{100}$ *of* $68.75, *which is* $12.37\frac{1}{2}$.

68.75
.18
55000
6875
12.3750

3. What is the interest of $175, for 1 year, at 7 per cent.? and what for 4 years? Ans. $12.25.—$49.00.

4. What is the interest of $175.45, for 3 years, at 7 per cent. per annum? Ans. $36.84.

5. What is the interest of 348.37\frac{1}{2}$, for 2 years, at 5 per cent. per annum? Ans. 34.83\frac{3}{4}$.

6. What is the interest of $157.89, for 5 years, at 5 per cent.? Ans. 39.47\frac{1}{2}$.

7. What is the interest of $74 for 4 years, at 7 per cent.? Ans. $20.72.

8. What is the interest of $100 for 1 year, at 6 per cent.? and what will the debt amount to?

Ans. Int. $6.—Am. $106.

9. What is the interest on the principal of $337 for 1 year, at 6 per cent.? and what is the amount of principal and interest? Ans. Int. is $20.22; and am. $357.22.

10. What is the interest and amount of $550 for 3 years, at 6 per cent.? Ans. Am. $649.

11. What is the amount of $387.39, for 5 years, at 6 per cent.? Ans. $503.60.

12. What is the amount of $1765 for 3 years, at 5 per cent.? Ans. $2029.75.

13. What is the amount of $65.48 for 2 years, at 7 per cent.? Ans. $74.64.

14. What is the interest of $85 for 2 months, at 6 per cent. per annum?

Ans. As 2 months is the $\frac{1}{6}$ part of a year, the per cent. for 2 months must be $\frac{1}{6}$ as much as the per cent. for a year. Therefore, the per cent. for 2 months is 1 per cent. This is more plainly to be seen when we speak of the interest. For if the interest on a dollar for a year is 6 cents, it is at the rate of 1 cent for every two months. Now, $\$85 \times .01 = .85$. The answer is 85 cts.

15. What is the interest of $194 for 8 months, at 6 per cent. per annum?

Ans. As the interest is 1 per cent. for two months, it will be as many times 1 per cent. as there are two months. In this question there are 4 times 2 months. Therefore, for 8 months, the interest is 4 per cent. $\$194 \times .04 = \$7.76$, the interest.

This principle will assist us in performing the following sums. For we have only to recollect that *the number per cent. is just half as much as the number of months.*

16. What is the interest of $379 for 14 months, at 6 per cent. per annum? Ans. $\$379 \times .07 = \$26.53$.

17. What is the interest of $849 for 21 months, at 6 per cent.? Ans. $\$849 \times .10\frac{1}{2} = \$89.14\frac{1}{2}$.

18. What is the interest of $154.78 for 1 year 9 months, at 6 per cent.? [1 y. 9 mo. = 21 mo.] Ans. $16.25.

19. What is the interest and amount of $457 for 1 year 6 months, (or 18 months,) at 6 per cent.? Ans. $41.13.

20. What is the amount of $35 for 2 years 3 months, at 6 per cent.? Ans. $\$39.72\frac{1}{2}$.

21. What is the amount of $578 for 2 years 5 months, at 6 per cent.? Ans. $661.81.

22. What is the amount of $36.87 for 2 years 8 months, at 6 per cent.? Ans. $42.77.

23. What is the interest on $48 for 1 month, at 6 per cent. per annum? Ans. $48 × .00½ = .24 cts.

24. What is the interest of $300 for 1 month, at 6 per cent. per annum? Ans. $1.50.

As banks, in their charters, are authorized to charge interest at the rate of 1 per cent. for 60 days, it has become a habit among mercantile men to do the same for small sums. Hence, the interest for 30 days is ½ per cent.; for 15 days it is ¼ per cent.; for 20 days it is ⅓ per cent.; for 10 days it is ⅙ per cent., &c. For another rule, see p. 246.

25. On the same principle, what is the rate per cent. for 12 days? For 6 days? For 40 days?

Ans. ⅕ per cent.; $\frac{1}{10}$ per cent.; $\frac{4}{10}$, or ⅔ per cent.

26. What is the rate per cent. for 45 days? For 50 days? For 24 days? For 36 days?

Ans. .00¾; .00⅝; .00⅖; .00⅗.

27. What is the rate per cent. for 18 days? For 42 days? For 48 days? For 54 days?

Ans. .00$\frac{3}{10}$; .00$\frac{7}{10}$; .00⅘; .00$\frac{9}{10}$. Or, as they may be written, .003; .007; .008; .009.

Hence, we derive the following

RULE

For finding the Rate per cent. for Months and Days.

Take half the even number of months for the number per cent., and if there remains an odd month, or any number of days, write $\frac{1}{60}$ of 1 per cent. for each day, (including the odd month if there is any,) reduce it as a vulgar fraction to its lowest terms, and annex it to the per cent. for the months.

Note.—If the fraction of two months can be reduced to *tenths* so as to be written as a decimal, that will be the

most convenient. This will always be the case, when the days can be divided by 6 without a remainder. Thus, 18 days = .3 per cent. written .003.

28. What is the rate per cent. for 21 days? Ans. .00$\frac{7}{10}$.

29. What is the rate per cent. for 1 mo. 19 days?
Ans. .00$\frac{19}{30}$.

30. What is the rate per cent. for 1 mo. 14 days?
Ans. .00$\frac{14}{15}$.

31. What is the rate per cent. for 1 mo. 26 days?
Ans. .00$\frac{13}{15}$.

32. What is the rate per cent. for 3 mo. 18 days?
☞ 3 mo. 18 d. = 2 mo. 48 days. Ans. .01$\frac{4}{5}$.

33. What is the rate per cent. for 5 mo. 10 days?

34. What is the rate per cent. for 9 mo. 12 days?

35. What is the rate per cent. for 13 mo. 6 days?

36. What is the rate per cent. for 1 yr. 3 mo. 24 days?

37. What is the interest of $374 for 5 mo. 18 days, at 6 per cent. per annum? [By the note under the rule.]

5 mo. 18 d. = 4 mo. 48 d.

6)48 days.
8 tenths per cent.

$374
.028 per cent. for 5 mo. 18 d.
2992
748
$10.472 Ans.

38. What is the interest of $876 for 11 mo. 9 days, at 6 per cent. per annum? (The per cent. is .05$\frac{13}{29}$.)
Ans. $49.49 +

39. What is the interest of $49.50 for 6 mo. 14 days, at 6 per cent. per annum? Ans. $1.60.

40. What is the interest of $75.54 for 9 mo. 20 days, at 6 per cent.? Ans. $3.65.

41. What is the interest of $15.12½ for 15 mo. 17 days, at 6 per cent.? Ans. $1.17.

42. What is the interest of $127.35 for 1 yr. 8 mo. 7 d., at 6 per cent.? Ans. $12.883.

43. What is the interest of $35.14 for 2 yrs. 9 mo. 15 d., at 6 per cent.? Ans. $5.895.

44. What is the interest of $53.41 for 3 yrs. 7 mo. 3 d., at 6 per cent.? Ans. $11.51.

45. What is the interest of $721.53 for 4 yrs. 1 mo. 18 days, at 6 per cent.? Ans. $178.94.

46. What is the interest of $217.15 for 3 yrs. 10 mo. 1 day, at 6 per cent.? Ans. $49.98.

47. What is the interest of $143.18 for 5 yrs. 2 mo. 29 days, at 6 per cent.? Ans. $45.077.

48. What is the interest of 18£. 15 s. [*reduced to decimals, as directed on page* 231. *The odd shilling* = .05.] for 2 yrs. 5 mo. 8 days, at 6 per cent.?
Ans. 2£. 14 s. 10¼ d.

49. What is the interest of 47£. 10 s. 6 d. for 1 yr. 4 mo. 24 days, at 6 per cent.? Ans. 3£. 19 s. 10 d.

50. What is the interest of 100£. 18 s. 9 d., for 3 years 5 months 2 days, at 6 per cent.? Ans. 20£. 14 s. 6 d.

51. What is the interest of 87£. 4 s. 7 d., for 2 years 7 months 27 days, at 6 per cent.? Ans. 13£. 18 s. 3 d.

52. What is the interest and amount of 44£. 18 s. 4½ d., for 3 years 3 months 10 days, at 6 per cent.?
Ans. Amt. 53£. 15 s. ¾ d.

53. What is the interest and amount of 78£. 4 s. 8 d., for 1 year 1 month 23 days, at 6 per cent.?
Ans. Amt. 83£. 12 s. 4½ d.

54. What is the amount of 81£. 9 s. 10 d., for 7 months 28 days, at 6 per cent.? [*See sum* 48.]
Ans. 84£. 14 s. 5¾ d.

55. What is the amount of 74£. 10 s. 9 d., for 1 year 3 months 11 days, at 6 per cent.? Ans. 80£. 5 s. 3½ d.

NOTE.—If the rate be 5 or 7 per cent. per year; after finding the interest at 6 per cent., if $\frac{1}{6}$ of it be subtracted, the remainder will be the interest at 5 per cent.; and if

$\frac{1}{6}$ of it be added, the sum will be the interest of it at 7 per cent.

56. What is the interest of $94.75 for 2 years 6 months 5 days, at 5 per cent.?—and at 7 per cent.?

94.75
.15 $\frac{1}{2}$ per cent. for 2 y. 6 m. 5 d.
47375
9475
789
14.2914 interest at 6 per cent.

6)14.29 interest, at 6 per cent.
2.38$\frac{1}{6}$ interest, at 1 per cent.

14.29 + 2.38$\frac{1}{6}$ = 16.67$\frac{1}{6}$ interest at 7 per cent.
14.29 − 2.38$\frac{1}{6}$ = 11.90$\frac{5}{6}$ interest at 5 per cent.

57. What is the interest and amount of $365 for 2 years 8 months 12 days, at 5 per cent.? Ans. Amt. 414.27\frac{1}{2}$.

58. What is the interest and amount of $650 for 3 years 10 months 21 days, at 7 per cent.? Ans. Amt. $827.07.

59. What is the amount of 47£. 15 s. for 2 years 11 mo. 13 days, at 5 per cent.? Ans. 54£. 15 s. 11$\frac{1}{2}$ d.

60. What is the amount of 94£. 1 s. 6 d. for 3 years 6 m. 6 days, at 7 per cent.? Ans. 117£. 4 s. 8 d.

61. What is the amount of $37.54 for 5 years 3 months 22 days, at 5 per cent.? Ans. $47.50.

62. What is the amount of $75.43 for 4 years 7 months 19 days, at 7 per cent.? Ans. $99.90.

63. A note of $320, with interest,* was given October 7, 1830, and paid July 11, 1832. What was the amount at the time of settlement? Ans. $353.81.

* The rate per cent. is rarely mentioned in the note. If it is given in New York, it is supposed to be 7 per cent.; if in Georgia, Alabama, Mississippi, or Louisiana, it is supposed to be 8 per cent. But if in other states, it is supposed to be 6 per cent. In this work it is 6 per cent. if no other is mentioned.

To subtract one date from another, set down the numbers of the given months, calling January 1, &c.; and then the number of days; and subtract, as in compound subtraction. Many, however, when calculating interest, find exactly how many *days* there are from one date to the other.

64. A note of $49.75, with interest, was given Jan. 4th, 1834, and paid June 7th, 1836. What was the amount at the time of payment? Ans. $56.988.

65. What is the amount of a note for $108, with interest, dated Feb. 12th, 1825, and paid March 1st, 1827? Ans. $121.30.

66. What is the amount of a note for $367.40, with interest, dated Nov. 8th, 1829, and paid March 4th, 1834? Ans. $462.68.

67. What is the amount of a note for $67.50, with interest, given 4th of March, and paid 27th of Dec., 1836? Ans. $70.79.

NOTE.—In order to obtain interest on notes, it is necessary to insert the words, *with interest*. If this is not inserted, the borrower pays only the face of the note. But if the note is given for any particular time, without interest, and is not taken up at that time, the creditor may receive interest from that time till the time of payment.

68. What is the amount of a note for $225, payable in 90 days, given June 6, 1829, and paid Jan. 7, 1830? [*The int. commences 90 days after June 6.*] Ans. $229.61.

69. A merchant bought goods to the amount of $1690, for which he gave his note, dated June 8th, 1829, on interest after 90 days. What was the amount of the note March 17th, 1831? Ans. $1845.19.

70. A merchant gave his note for $650, dated 4th of May, 1831, with interest after 90 days. What was the amount of it June 7th, 1832? Ans. $683.04.

71. What is the amount of a note for $673.37½, dated August 9th, 1827, with interest after 60 days, and paid September 7th, 1828? Ans. $710.30.

72. A man gave a note for $874, with interest, and 6 months and 14 days, from that time, paid $600; and 8 months 10 days, from that payment, paid the balance. How much was due after paying the $600? And what was the amount to at the time of settlement?

Ans. After paying $600, due $302.25. Due at time of settlement, $314.84.

73. On the 14th of December, 1827, a man gave a note of $678, with interest; and afterwards paid on the 8th of March, 1829, $175; on the 22d of August, 1829, $250; on the 17th July, 1830, $200; on the 14th September, 1831, $87. How much was due after each payment? And what was the balance on the 1st August, 1832?

Ans. Due, after paying $175, $553.17
" " " 250, 318.28
" " " 200, 135.52
" " " 87, 57.94
And, at the time of settlement, 61.00.

When the borrower pays a *part* of the debt, the sum that he pays is written on the back of the note, and is called an *endorsement*. In such cases, the method of computing the interest is different in different states. The best and the most general rule is that adopted by Chancellor Kent, of New York. It is as follows:

Rule

For casting interest, when partial payments have been made.

Apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal; and the subsequent interest is to be computed on the

balance of principal remaining due. But, if the payment be less than the interest, the interest must continue on the former principal, until other payments are made, which, in all, shall exceed the interest due at the time of the last payment; and then the surplus is to be applied towards discharging the principal. After which, interest is to be computed on the balance, as aforesaid.

BANK INTEREST.

The rate of interest allowed to banks, by law, is $\frac{1}{2}$ per cent. for 30 days; that is, they may estimate a *month* as 30 days. We have seen that, in calculating interest for days, this estimation is of great advantage; and, on this account, merchants have generally adopted it in their mercantile transactions.

Notes are generally given to banks for 30, 60, or 90 days, and sometimes longer. We therefore have the following

EASY RULES.

1. The interest for 60 days is 1 per cent., or $\frac{1}{100}$ of the principal. It is found by cutting off the last two figures.

2. The interest for 30 days is half as much as for 60 days.

3. The interest for 90 days is as much as the interest for 60 days and for 30 days put together.

It may be well to state that although a note is said to be drawn for 30, 60, or 90 days, yet the borrower is allowed *3 days' grace;* that is, 3 days longer than the time to pay it in. On this account, interest is computed on 33, 63, or 93 days.*

4. The interest on 3 days is $\frac{1}{10}$ as much as for 60 days.

* Banks in Philadelphia charge interest for 34, 64, or 94 days.

Example.

74. What is the interest on a note of $375 for 60 days; and for 30 days; and for 90 days?

$\frac{1}{240}$)3.75 int. for 60 d.	$\frac{1}{240}$)3.75 for 60.	$\frac{1}{240}$)3.75 for 60.
187 grace.	1.875 for 30.	1.875 for 30.
3.937 Ans.	187 grace.	5.625 for 90.
for 60 days.	2.062 Ans.	187 grace.
	for 30 days.	5.812 Ans.
		for 90 days.

75. What is the interest on a note of $137 for 30 days.

Ans. 75 cts.

The interest on a note to a bank is taken from the principal, when the borrower receives the money; and then it is called *discount*. In the case of the note last spoken of, the borrower would receive $136.25 instead of $137.

76. How much money would I obtain by getting a note of $875 discounted at the bank for 90 days?

Ans. $861.43¾.

77. How much money would I obtain by getting my note for $360 discounted for 60 days. Ans. $356.22.

78. How much money would I receive on a note of $480 payable to a bank in 30 days? Ans. $477.36.

79. What discount would be taken by a bank from a note of $890, payable in 90 days? Ans. $13.79½.

80. What would a bank charge for discounting a note of $375, payable in 30 days? Ans. $2.06¼.

81. I offered a note of $975 for 60 days to a bank. What did I receive for it? Ans. $964.76¼.

82. What is the discount, in one of the Philadelphia banks, on a note of $135, payable in 90 days? (*The interest for the 4 days' grace is $\frac{1}{15}$ of the interest for 60 days.*)

Ans. $2.11¼.

83. What would I obtain from a Philadelphia bank, for a note of $875, payable in 60 days? Ans. $865.66⅔.

84. If when the note last mentioned comes due, I wish to keep the money longer, what must I pay to have it discounted for 30 days more? Ans. $4.96.

From the preceding principle, business men have derived

A SHORT RULE

For calculating Interest for a small Number of Days.

1. Find the interest for 60 days by cutting off the last two figures as before.

2. Then the interest for the required number of days will be so many *sixtieths* of the interest for 60 days.

Operation.—Cut off two figures, then multiply by the number of days, and divide by 60. The answer will be in cents.

85. What is the interest on $140 for 27 days?

140
27
980
280
60)378.0
.63 cts.

86. What is the interest on $420 for 110 days?
Ans. $7.70.

This, it will be seen, is nearly the same as the rule given on page 238.

Though the foregoing methods are generally adopted by merchants, yet, as it is charging 6 per cent. for 360 days instead of 365 days, some persons object to it. And with large sums, the difference is well worth noticing. For instance, the interest of $100 for one year, when found by multiplying by 365, and divided by 60, is $6.08⅓, instead of $6, the true interest.

But, by a little attention, the foregoing rules will be of great use in finding the true interest. For, if 360 days will give the true interest of a year, 365 days will give the interest for 5 days too much. This is equal to $\frac{5}{360}$ too much. But $\frac{5}{360} = \frac{1}{72}$. Therefore the interest calculated for 365 days, in this manner, is $\frac{1}{72}$ too much. In proof of this, it will be found that $\frac{1}{72}$ of \$6.08$\frac{1}{3}$ is 8$\frac{1}{3}$ cts. Now if the interest calculated in this manner for a year, is $\frac{1}{72}$ too much, so it will be $\frac{1}{72}$ too much when calculated for any other time. Hence, we derive the following

RULE.

Calculate the interest at the rate of 1 per cent, for 60 days; and then subtract from the interest thus found, $\frac{1}{72}$ of itself.

87. What is the interest on \$420 for 90 days?

4.20 for 60 d.
2.10 for 30 d.
$\frac{1}{72}$) 6.30 for 90 d.
86
6.2114 true int.

88. The pupil may commence at the 74th question, and find the true interest for all the sums to the last.

COMPOUND INTEREST.

EXPLANATION AND RULES.

WHEN money has been kept by a debtor during a year, the creditor has a right to demand the interest, or to require the debtor to give a new note for the amount instead of the old one. If the debtor takes the old note and gives a new one, he *renews* the note. This is done at the bank every 60 or 90 days. But in common business, it is frequently

the case that the note is not renewed; but by agreement, the interest at the end of the year is added to the principal, and the interest for the ensuing year cast on the amount. Thus, a note may run for several years without being renewed, but at the close of the time be calculated so as to give the same amount that it would in case of its yearly renewal. This is called *Compound Interest.* It may be well to state that in several of the states, compound interest is not authorized by law.

RULE 1.—The operation may be performed as in simple interest, with the exception of adding the interest at the end of the year, and making the amount a new principal for the coming year.

RULE 2.—But a more expeditious method is to find the amount of 1 dollar for the time, and to multiply this amount by the given number of dollars in the question. Thus,

1.00	
.06	
.0600	interest for 1 yr.
1.00	
1.06	amount for 1 yr.
.06	
.0636	interest for the 2d yr.
1.06	
1.1236	amount for the 2d yr.
.06	
.067416	interest for the 3d yr.
1.1236	
1.191016	amount for 3 yrs.

Now it is evident, that, if 1 dollar will amount to $1.191016 in 3 years, $5 will amount to 5 times as much, and so with any other principal.

After finding the amount for the *years*, the interest for *months* and *days*, if there be any, is to be found by simple interest rules.

APPLICATION.

1. WHAT is the amount of a note for $871, at 6 per cent. compound interest, supposing that it has run 4 years 7 mo. 18 days?

[1st method.]

871
.06
52.26 int.
871
923.26 am. 1
.06
55.3956 int.
923.26
978.65 am. 2.
.06
58.7190 int.
978.65
1037.37 am. 3.
.06
62.2422 int.
1037.37
1099.61 am. 4.
.038 rate for 7 mo. 18 d.
879688
329883
41.78518 int.
1099.61
1141.395 whole amount.

For the accommodation of those who wish to work by the second method, the following table has been formed; in which the amount of 1 dollar or 1 £. is already found for any number of years from 1 to 10. It is followed by directions for using it.

A TABLE

Showing the Amount of 1 *dollar or* 1£. *on Compound Interest for any Number of Years and Parts of Years.*

Yr.	5 per cent.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	4½ per cent.
1	1.050000	1.055000	1.060000	1.06500	1.07000	1.04500
2	1.102500	1.113025	1.123600	1.13422	1.14490	1.09242
3	1.157625	1.174241	1.191016	1.20794	1.22504	1.14116
4	1.215506	1.238824	1.262477	1.28645	1.31079	1.19252
5	1.276281	1.306950	1.338225	1.37007	1.40254	1.24618
6	1.340095	1.378842	1.418519	1.45912	1.50071	1.30226
7	1.407100	1.454679	1.503630	1.55396	1.60576	1.36086
8	1.477455	1.534686	1.593848	1.65496	1.71836	1.42210
9	1.551328	1.619094	1.689479	1.76253	1.83864	1.48609
10	1.623894	1.708144	1.790847	1.87709	1.96734	1.52297
Mo.						
1	1.00416	1.00458	1.00500	1.00541	1.00583	1.00375
2	1.00833	1.00916	1.01000	1.01083	1.01166	1.00750
3	1.01250	1.01375	1.01500	1.01625	1.01750	1.01125
4	1.01666	1.01833	1.02000	1.02166	1.02333	1.01500
5	1.02083	1.02291	1.02500	1.02708	1.02916	1.01875
6	1.02500	1.02750	1.03000	1.03250	1.03500	1.02250
7	1.02916	1.03208	1.03500	1.03791	1.04083	1.02625
8	1.03333	1.03666	1.04000	1.04333	1.04666	1.03000
9	1.03759	1.04125	1.04500	1.04875	1.05250	1.03375
10	1.04166	1.04583	1.05000	1.05416	1.05833	1.03750
11	1.04584	1.05042	1.05500	1.05959	1.06416	1.04125
Da.						
1	1.000139	1.000153	1.000166	1.000180	1.000194	1.000125
2	1.000277	1.000305	1.000333	1.000361	1.000388	1.000250
3	1.000416	1.000458	1.000500	1.000541	1.000583	1.000375
4	1.000555	1.000611	1.000666	1.000722	1.000777	1.000500
5	1.000694	1.000764	1.000833	1.000903	1.000972	1.000625
6	1.000833	1.000916	1.001000	1.001083	1.001166	1.000750
7	1.000971	1.001069	1.001166	1.001263	1.001359	1.000875
8	1.001110	1.001221	1.001333	1.001444	1.001554	1.001000
9	1.001250	1.001375	1.001500	1.001635	1.001750	1.001125
10	1.001388	1.001528	1.001666	1.001805	1.001944	1.001250
11	1.001528	1.001681	1.001833	1.001885	1.002138	1.001375
12	1.001666	1.001833	1.002000	1.002166	1.002333	1.001500
13	1.001805	1.001986	1.002166	1.002346	1.002427	1.001625
14	1.001994	1.002139	1.002334	1.002528	1.002622	1.001750
15	1.002083	1.002291	1.002500	1.002708	1.002916	1.001875

To find the amount of any sum for years, months, and days; first find the amount for the years, according to Rule 2, by the table.

After the amount for the years has been found, find the amount of *that amount* for the remaining time. This is done by adding the decimals only, (which, without the integer 1, represents the interest,) that are opposite the given month and days, and afterwards prefixing the 1 to this sum. If the given days be more than 15, add to the decimal opposite 15, the decimal opposite such other number as, added to 15, will make the given number; as, in the following example:

2. What is the amount of $375.14 for 3 yrs. 7 mo. 9 ds., at 6 per cent. compound interest?

Amount of 1 dollar for 3 years = 1.191016

		375.14	
		4764064	
		1191016	
		5955080	
Int. of $1 for 7 mo.	= .0350	8337112	
" " 9 d.	= .0015	3573048	
	.0365	446.7797	Am. for 3 yrs.
	1.00	1.0365	
Am. for 7 mo. 9 d.	= 1.0365	2233985	
		2680782	
		1340391	
		446797	
		463.1050905	

3. What is the amount of $476.17, on compound interest, for 4 yrs. 9 mo. 17 d., at 6 per cent.? Ans. $629.90.

4. What is the amount of a note for $761.49, on compound interest, during 5 yrs. 8 mo. 28 d., at 6 per cent.? Ans. $1064.56.

5. What is the amount of a note for $97.125, on compound interest, during 6 yrs. 10 mo. 29 d., at 5½ per cent.? Ans. $140.65.

6. What is the amount of $901.45, on compound interest, during 5 yrs. 25 d., at 7 per cent.? Ans. $1270.46.

7. What is the amount of $147.18, on compound interest, during 4 yrs. 1 mo. 19 d., at 6½ per cent.? Ans. $191.01.

8. What is the amount of $159.50, on compound interest, during 5 yrs. 2 mo. 4 d., at 7 per cent.? Ans. $226.48.

9. What is the amount of $311, on compound interest, for 4 yrs. 6 mo. 20 d., at 6 per cent.? Ans. $405.71.

10. What is the amount of $146.47, on compound interest, for 2 yrs. 7 mo. 27 d., at 7 per cent.? Ans. $175.42.

11. What is the amount of $189.75, on compound interest, during 3 yrs. 1 mo. 16 d., at 5½ per cent.? Ans. $224.37.

12. What is the amount of $478, on compound interest, during 2 yrs. 10 mo. 22 d., at 6½ per cent.? Ans. $573.67.

DISCOUNT.

1. WHAT is the discount of a note for $75, at 60 days, at 6 per cent.?

2. What is the discount of a note for $178, at 90 days?

3. What is the discount of a note for $344, at 30 days?

4. A merchant had sugar, for which he asked $10 a hundred; but offered to sell it, for cash, for 12 per cent. discount. What did he take from the price; and for what did he sell it?

5. I bought a book for $2.25, but for cash a discount of 25 per cent. was made. What did I give for the book?

6. A merchant bought a quantity of shingles, at $4 a bundle, but was told, if he would purchase 100 bundles, there should be a discount of 20 per cent. If he agreed to that, what did he give for each bundle?

7. A merchant had wine that he sold for $1.75 a gallon, retail, but made a discount of 15 per cent. to those who purchased 25 gallons. What did they give a gallon?

8. Bought a quantity of glass ware for $47.18¾, but for cash a discount of 12½ per cent. was made. What did I give for the glass?

9. Bought a quantity of cloth, marked at $6 a yard, but had a discount of 33⅓ per cent. What did it cost me a yard?

10. Bought a hogshead of sugar, sold for $14 a hundred, but had a discount of 25 per cent. What was the cost of it a hundred?

11. What must be the discount for the present payment of $128, due 4 months hence? And what must the payment be?

12. What is the discount for the present payment of $425, due 5 months 10 days hence?

Note.—It has been stated before, that a note due at any future time, is worth only the face of it at that time, unless it reads *with interest*. If such notes are paid before they become due, the payer pays so much only as, *with interest* to that time, will amount to the given debt. This, in small sums, is generally found by subtracting the interest of the given sum from the sum itself. The remainder of the sum to be paid is called the *present worth*.

13. What is the present worth of $1000, due 26 days hence, at a discount of 6 per cent.?

14. What is the present worth of $896, due 1 month 10 days hence, at 7 per cent. interest? Ans. $889.05.

15. What is the present worth of $575, due 2 months 18 days hence, at 6½ per cent. interest? Ans. $566.90.

16. What is the present worth of $350, due 3 months 27 days hence, at 6 per cent. interest? Ans. $343.17½.

17. What is the discount of $425, due 2 months 17 days hence, at 7 per cent.? Ans. $6.36.

18. What is the discount of $176, due 3 months 14 days hence, at 6 per cent.? Ans. $3.05.

19. What is the present worth of $300, due 6 months hence, at 6 per cent. interest? Ans. $291.

Note.—Thus it appears, that if a person pays at this time a debt of 300 dollars, due 6 months hence, he pays but 291 dollars. But we find, if 291 dollars were put at interest for 6 months, at 6 per cent., it would amount to $299.73, which is 27 cents less than the debt. This is occasioned by calculating the interest on the *amount*, rather than on the *principal*; and, by so doing, the payer receives interest on the interest, which is not strictly just. This is however, done in ordinary business; and in small sums, for short periods, does not amount to much.

But there is a method by which we can find the exact present worth. Thus, the amount of 100 dollars, for 1 year at 6 per cent. is $106, as follows: $100 \times 1.06 = 106$. Hence, we know that $106 \div 1.06 = 100$; and derive the knowledge that any sum, divided by the amount of 1 dollar for any time, will show the present worth of that sum, due for the same time.

Rules

For finding the Discount on any Sum due at a future Time.

Rule 1.—Calculate the interest on the given sum, at the given rate and time. This will be the discount, which, subtracted from the given sum, will show the present worth, as *generally* found: or,

Rule 2.—Divide the given sum by the amount of 1 dollar at the given rate and time, and the quotient will be the present worth of the given sum, as *accurately* found.

APPLICATION OF RULE 2.

1. What is the present worth of $225, due 4 months 14 days hence, when interest is 6 per cent.? Ans. $220.09.

2. What is the present worth of $408, due 6 months 7 days hence, when interest is 7 per cent.? Ans. $393.68.

3. What is the present worth of $139, due 2 months 6 days hence, when interest is 6 per cent.? Ans. $137.48.

4. What is the present worth of $48, due 1 month 24 days hence, when interest is 7 per cent.? Ans. $47.50.

5. What is the present worth of $90, due 2 months 17 days hence, when interest is 6 per cent.? Ans. $88.86.

6. What is the present worth of $405, due 1 month 3 ds. hence, when interest is 7 per cent.? Ans. $402.42.

DUTIES.

DUTIES are sums paid to the government on the importation of foreign merchandise. In some cases, they are *specific*; that is, laid at a certain rate for a bushel, a ton, a gallon, a yard, &c.

In other cases, they are *ad valorem*; that is, laid according to the value; or more properly speaking, at so much per cent. on their actual cost.

1. What is the duty on a quantity of hemp, invoiced at $495; at 14 per cent.? Ans. $69.30.

2. What is the duty on a quantity of books, invoiced at $376.50; at 20 per cent.? Ans. $75.30.

3. What is the duty on a quantity of wine, invoiced at $4975; at 45 per cent.? Ans. $2238.75.

4. What is the duty on a quantity of books, invoiced at 345£. 10 s. 6 d.; at 20 per cent.?

Note.—Custom-house officers first change sterling money to federal currency, by multiplying by 4.80.

Ans. 345£. 10 s. 6 d. = £345.525; which × 4.80 = $1658.52. The duty is $331.70.

5. What is the duty on a quantity of sugar, invoiced at 878£. 14 s. 10 d.; at 18 per cent.? Ans. $759.23.

6. What is the duty on a quantity of salt, invoiced at 625£.; at 42 per cent.? Ans. $1260.

TAXES.

Taxes are sums paid by individuals for the support of the government.

A tax is either a certain sum on each individual, called a *poll tax*; or, so much per cent on the property of the people. In some states, there is no poll tax.

1. What is the tax on an assessment of $4647, at the rate of $2\frac{1}{2}$ cents on a dollar?

Note.—$2\frac{1}{2}$ cts. on a dollar is the same as $2\frac{1}{2}$ per cent.

Ans. $116.17\frac{1}{2}$.

2. What is the tax on $875.34 at $3\frac{1}{4}$ cts. on a dollar?

Ans. $28.45.

3. What is the tax on $7840 at $2\frac{1}{2}$ cts. on a dollar?

Ans. $196.

Assessors generally make a table, containing a tax on $1, $2, &c. up to $9; then on $10, $20, &c. up to $90; then on $100, $200, &c. up to $1000. Then, by knowing any one's property, his tax may be taken from the table without the trouble of calculation.

4. A town valued by inventory, at $10,204,046, wishes to raise a tax of $127550. What must it levy on a dollar?

Ans. $1\frac{1}{4}$ cents.

EQUATION OF PAYMENTS.

DEFINITION.

When several payments are due at different times, and you ascertain one time when all may be paid, so that neither party is a loser of interest; the operation is called *Equation of Payments.*

Rule.

Multiply each payment by the time when it is to be paid, add all the products together, and divide the sum of them by the whole debt; and the quotient will be the equated time, or the proper time for paying the whole sum.

APPLICATION OF THE RULE.

1. A merchant has due to him the following sums from one man: $300, payable in 2 months, $200 in 3 months, $150 in 4 months. They both agree to make it all in one payment. When shall it be done?

With the interest of $300 for 2 months, the interest of 1 dollar could be paid for 2 months, and then again for 2 months, and so on for 300 times. Hence,

Int. of $300 *for* 2 *months* = *int. of* $1 *for* 600 *months.*
" 200 " 3 " = " 1 " 600 "
" 150 " 1 " = " 1 " 600 "

Therefore it all = *int. of* $1 *for* 1800 *months; which* = *the interest of* $650 *for* $\frac{1}{650}$ *of* 1800 *months, which* = 2 *mo.* $23\frac{1}{13}$ *days.*

2. A has 5 notes of D, viz., 400 dols., due in 10 days; 300 dols., due in 20 days; 500 dols., due in 30 days; 200 dols., due in 60 days; and 250 dols. due in 90 days. If D makes it all in one payment, when must it be made?

Ans. 36 days.

3. B owes C $380; of which 100 is to be paid in 6 months, 120 in 7 months, and 160 in 10 months. As they agree that the whole shall be paid at one time, when must it be? Ans. At 8 months.

4. A merchant sold goods amounting to $3000, to be paid as follows: $500 in 2 months, $1000 at 5 months, and $1500 in 8 months. But if they make one payment, at what time must it be? Ans. 6 months.

5. Y owes Z $1200; of which 240 are to be paid now, 480 in 5 months, and the balance in 10 months. What would be the equated time of paying the whole?

Ans. 6 months.

6. A merchant sold goods, amounting to $1200, payable in 6 months; but, as he wanted money, the purchaser agreed to pay him $500 cash, if he would wait a proportionable time for the balance. At what time would the balance become due?

$1200 for 6 months = $1 for 7200 months, which, divided by $700 the balance; = 10 months 8$\frac{4}{7}$ days.

Equation of Dividends, or Fellowship.

7. A, B, and C traded with the following capitals: A put in $200 for 6 months, B put in 800 dollars for 8 months, and C put in 700 dollars for 9 months. They gain 600 dollars. What is each man's share?

$200 for 6 months	=	$1200 for 1 month	
800 " 8 "	=	6400 " " "	
700 " 9 "	=	6300 " " "	
Whole capital	=	$13900 for 1 month.	

A's share is $\frac{1200}{13900}$
B's " $\frac{6400}{13}$
C's " $\frac{6300}{13900}$
} of $600 gain, or
$51.80
276.26
271.94
600.00

8. A, B, and C traded in partnership, and gained $769. What was each man's share of the gain, supposing A put in $500 for 1 year; B, $450 for 10 mo.; and C, $600 for 8 mo.? Ans. A, 301.56\frac{14}{31}$; B, 226.17\frac{11}{17}$; C, 241.25\frac{15}{31}$.

9. A, B, and C traded and lost $436. What was each man's share of the loss, supposing A put in $400 for 6 months; B, $250 for 9 months; and C, $500 for 7 mo.?

Ans. A, $128.39; B, $120.36; C, $187.24.

10. A, B, and C were in partnership one year. At first, A put in $1000; B, $1200; and C, $900. But 4 months after, A took out $100, and C put in $100; and 3 months from that time, A took out $100, and B put in $200. Their whole gain was $2875. What was each man's share of it?

Ans. A's share, $815.98; B's, $1174.40; C's, $884.62.

11. D and E entered into partnership for 2 years, and commenced with a capital of $1200 each. In 6 months, D put in $100 more, and in 6 months from that time put in $150 more. In one year from the commencement of their business, E put in $200, and 6 months from that time he wishes to put in so much, that, when the time expires, their dividend shall be equal. How much must he put in?

Ans. $200.

D's, $1200 for 24 months = $28800 for 1 month.
" 100 " 18 " = 1800 " 1 "
" 150 " 12 " = 1800 " 1 "
D has put in $32400 for 1 month.

E's, $1200 for 24 months = $28800 for 1 month.
" 200 " 12 " = 2400 " 1 "
E has put in $31200 for 1 month,
and to equal D, must put in 1200 for 1 month;
which, divided by 6 months, the remaining time, = $200.

* The fractions to be reduced.

12. Y and Z trade in partnership, and gain $1285. At first, Y put in $1500, and Z, $1700. Three months from that time, Y put in $100, and Z, $75. Four months afterwards, Y took out $375. After they had traded a year, they found that they had gained $1285. What was each man's share of it?

Ans. Y's share was $574.20; and Z's, $710.79.

Equation of Prices, or Alligation Medial.

13. A merchant had 3 sorts of wine, of different prices, which he mixed as follows: 10 gallons, at $1 a gallon; 8 gallons, at $1.25 a gallon; and 12 gallons, at $1.37½ a gallon. What is one gallon of this mixture worth?

10 gallons, at $1.00 a gallon = $10.00
8 " 1.25 " = 10.00
12 " 1.37½ " = 16.50
30 gallons, at different prices = $36.50
and 1 gallon = $ 1.21⅔ Ans.

14. A merchant mixed teas, of three different qualities, as follows: 18 lbs., at 97 cts. a lb.; 20 lbs., at $1.12½ a lb.; and 15 lbs., at $1.15 a lb. What is the worth of 1 pound of this mixture? Ans. 1.07\frac{50}{53}$.

15. A miller mixed grain, for provender, as follows: 20 bushels of rye, at 75 cents a bushel; 15 bushels of corn, at 50 cts. a bushel; 24 bushels of oats, at 32 cts. a bushel; and 4 bushels of flax-seed, at 62½ cts. a bushel. What is his provender worth a bushel? Ans. 51$\frac{5}{13}$ cts.

16. A brewer mixed beer as follows: 100 gallons, worth $1 a gallon; 200 gallons, worth $1.12½ a gallon; and 50 gallons, worth 87½ cts. a gallon. What is the mixture worth a gallon? Ans. 1.05\frac{5}{14}$.

17. A grocer mixed sugars as follows: 2 hundred, worth $8 a C.; 4 C. worth $10; 3 C. worth $7½; and 1 C. worth $12½. What is the worth of this mixture by the pound?

Ans. 9$\frac{1}{16}$ cts.

EQUATION OF QUANTITIES, OR ALLIGATION ALTERNATE.

INDUCTIVE EXERCISES FOR THE SLATE.

1. A grocer has two kinds of sugar, one at 7 cents a pound, and the other at 12 cents a pound. How much of the dearest is worth just one cent more than an equal quantity of the cheapest?

Ans. As a whole pound of one is worth *five cents* more than a whole pound of the other, $\frac{1}{5}$ of a pound of the dearest is worth *one cent* more than $\frac{1}{5}$ of a pound of the cheapest.

2. How, then, can he make the cheapest one cent dearer for a pound?

Ans. Take out $\frac{1}{5}$ of a pound, and put in $\frac{1}{5}$ of a pound of the dearer.

3. How can he make the cheapest 2 cents dearer for a pound?—3 cents dearer?—4 cents dearer?

4. Suppose a grocer has one kind of tea that is worth 75 cents a pound, and another kind that is worth $1.00 a pound. How much of the dearest is worth *one cent* more than an equal quantity of the cheapest?

5. How can he mix a pound to be worth 1 cent more than the cheapest?—3 cents more?—4 cents more?—5 cents more?—6?—10?—12?—15?—20?

6. How can he mix a pound to be worth 3 cents *less* than the dearest?—5 cents less?—8 cents less?—11 cents?—14?—16?—18?—24?

7. Supposing there are two kinds of raisins, priced at 14 cents and 20 cents; how can you mix a pound of them so that it shall be worth 16 cents?

8. What number becomes the *denominator* in the fraction of a pound? Ans. *The difference of the two original prices.*

9. What does the numerator of the fraction signify?

Ans. The part taken from the quantity, and which is *to be replaced by so much of the other quantity.*

10. Supposing you have corn at 42 cts. a bushel, and rye at 54 cts. a bushel; what change must you make with the cheapest to obtain a mixture worth 50 cts. a bushel?—How much of each kind will there be?

11. What change must you make with the dearest to obtain a mixture worth 50 cts. a bushel?—How much of each kind will there be?

12. What may we learn from the fact that both the 10th and the 11th questions lead to the same result?

Ans. *That the difference between the least price and the required price, represents the quantity taken of the dearest kind; and the difference between the highest price and the required price, represents the quantity taken of the cheapest kind.*

As in such operations we connect the two different prices, the rule has been called *Alligation*, which means *tying together.*

DEFINITION.

ALLIGATION is a method of finding what quantities of different ingredients may be taken to make a mixture of a required price.

RULE.

First, Place the required prices under each other, and the mean price at their left. And then connect or link a price that is less than the mean, with a price that is greater, until every price is connected with another.

Second, Find the difference between the given price and the mean price, and place that difference opposite the price with which it is connected.

Third, The difference opposite a given rate, or, if more than one, the sum of the differences will be the quantity belonging to that rate.

APPLICATION.

1. A grocer would mix several qualities of sugar, viz. at 10 cts., 13 cts., and 15 cts. a lb. What quantity of each sort must he take to make a mixture worth 12 cts. a lb.?

Ans. *He must take* $\frac{2}{3}$ *of a pound from the* 10 *to put with the* 13; *and* $\frac{1}{3}$ *from the* 13 *to put with the* 10; *or, which is the same,* 2 *lb. of the* 13 *and* 1 *lb. of the* 10. *He must also take* $\frac{2}{3}$ *from the* 10 *to put with the* 15, *and* $\frac{2}{3}$ *from the* 15 *to put with the* 10; *or, which is the same, &c.*

	cts.		lb.	
cts. 12	10	1 + 3 =	4	Ans.
	13		2	
	15		2	

2. A grocer has several sorts of sugar; some at 14 cts., some at 13 cts., some at 11 cts., and some at 10 cts. a lb. How much of each sort will make a mixture worth 12 cts. a pound?

Ans. 2 lb. at 14 cts., 1 lb. at 13 cts., 1 lb. at 11 cts., and 2 lb. at 10 cts. Or, 1 lb. at 14 cts., 2 lb. at 13 cts., 2 lb. at 11 cts., and 1 lb. at 10 cts.; because the prices may be linked in two different ways.

3. A grocer has two sorts of tea; viz. at 9s. and at 15s. a lb. How must he mix them so as to afford the composition for 12s. a lb.? Ans. An equal quantity of each sort.

4. A grocer has three sorts of tea; viz. at $1 a lb., at 66 cts. a lb., and at 50 cts. a lb. How can they be mixed so as to be sold for 75 cts. a lb.?

Ans. 25 lb. of that for 50 cts., 25 lb. of that for 66 cts., and 34 lb. of that for $1.

5. A merchant has 28 lb. of pepper, at 3s. a lb., which he wishes to mix with other pepper at 2s. 1d. and at 1s. 5d., so as to sell the mixture at 2s. 4d. a lb. How much of each of the other sorts must he take?

2s. 4d. = 28d.	3s. = 36d.	3 +	11 = 14
	2s. 1d. = 25d.		8
	1s. 5d. = 17d.		8

The answer is 14 lb. of that for 3s., and 8 lb. of each of the others; but of the first, he has 28 lb., which is twice as

much; therefore there must be twice as much as 8 lb. of each of the other two sorts. Ans. 16 lb. of each.

6. A grocer wishes to mix 9 lb. of sugar, worth 6 cts. a lb., with two sorts, worth 7 and 11 cts. a lb.; so as to sell it for 9 cts. a lb. How much of those sorts must he use?

The answer, by linking, would be, 2 lb. at 6 cts. But there must be $\frac{9}{2}$ of that quantity. So that there will be 9 lb. at 7 cts., and $22\frac{1}{2}$ lb. at 11 cts.

7. A grocer has 37 lb. of sugar, worth 16 cts. a lb. But as it is a little too dear, he wishes to mix it with other sorts, worth 10 cts. and 13 cts. a lb.; so as to sell it for 12 cts. a lb. How much of the other sorts must he use?

Ans. $92\frac{1}{2}$ lb. of that for 10 cts., and 37 lb. of that for 13 cts.

8. A goldsmith has 6 oz. of gold of 24 carats fine, which he wishes to mix with other kinds, 19, 21, and 24 carats fine, so that the compound may be 22 carats fine. How much of the other kinds must he take? Ans. 6 oz. of each.

9. A farmer mixed 15 bushels of rye, worth 64 cts. a bushel, with Indian corn worth 56 cts. a bushel, and oats worth 28 cts. a bushel, in such a manner that the mixture was worth 47 cts. a bushel. How much of the corn and oats did he take?

Ans. 15 bu. of the Indian corn, and $20\frac{10}{19}$ bu. of the oats.

10. A grocer has three sorts of sugar, at 4d., 6d., and 11d. a lb. But he has an order for 28 lb. at 7d. a lb. How much of each sort must he put into a mixture, to accommodate the order?

Ans. By linking the prices, we find 4 lb. of each sort, which would amount to 12 lb. But there must be $\frac{28}{12}$ as much. Therefore there must be of each sort 9 lb. $5\frac{1}{3}$ oz.

11. A farmer wishes to mix rye worth 54 cts., Indian corn worth 50 cts., oats worth 37 cts., and flaxseed worth 75 cts. a bushel; so as to make 20 bushels, worth 45 cts. a bushel. How much of each kind must he use?

Ans. $2\frac{6}{17}$ bushels each, of rye, corn, and flaxseed; and $12\frac{16}{17}$ bushels of oats.

RATIO AND PROPORTION;

OR,

RULE OF THREE.

INDUCTIVE EXERCISES FOR THE SLATE.

1. When we compare one number with another, in order to see what part of one the other is, the operation is called *ratio*.

2. When I ask what is the ratio of 7 to 5, I mean to inquire how 7 compares with 5; and the answer is $\frac{7}{5}$. So in the foregoing questions, the pupil must remember that the object is to find how the *first* number compares with the *last*. Therefore the *first* number must be the numerator.

3. What is the ratio of 9 to 12? Of 4 to 7? Of 8 to 16? Of 5 to 25? Of 12 to 6? Of 30 to 5?

4. Of what is $\frac{7}{6}$ the ratio? Ans. 7 to 6.

5. Of what is $\frac{4}{9}$ the ratio? $\frac{8}{7}$? $\frac{5}{4}$? $\frac{7}{14}$? $\frac{6}{3}$? $\frac{8}{16}$?

6. What is the ratio of 4 to 8? Ans. $\frac{4}{8}$, which $= \frac{1}{2}$.

7. What is the value of the following ratios? $\frac{6}{3}$, $\frac{9}{3}$, $\frac{6}{12}$, $\frac{10}{44}$, $\frac{18}{60}$, $\frac{16}{40}$, $\frac{10}{12}$.

8. Hence, it seems that ratios can be equal to one another, even if they are not expressed by the same figures. For, as $\frac{3}{4} = \frac{12}{16}$, so the ratio of 3 to 4 = the ratio of 12 to 16; and as $\frac{5}{3} = \frac{15}{9}$, so the ratio of 5 to 3 = the ratio of 15 to 9.

9. Now mention 6 other ratios equal to the ratio of 4 to 6? 5 to 15? 9 to 81? 4 to 40? 3 to 7? 1 to 8?

10. To denote an equality of the ratios, they may be written thus,

11 : 21 = 33 : 63, or, 11 : 21 :: 33 : 63; which is read, 11 *is to* 21, *as* 33 *is to* 63.

11. Of these, the first and the last terms are called *extremes*, and the two middle terms are called *means*; and all the four terms put together are called a PROPORTION.

12. Now suppose we wish to inquire whether the ratio of 11 to 21 is equal to the ratio of 33 to 63. We will write them as fractions, and bring them to a common denominator.

$$\frac{11}{21} \text{ \& } \frac{33}{63} = \frac{693}{1323} \text{ \& } \frac{693}{1323}.$$

13. In this operation, we multiplied 11 and 63 together for *one* of the new numerators. These are the *two extremes*.

We also multiplied 21 and 33 together for the *other* new numerator. These are the *two means*.

14. If the product of the two means in a proportion be 24, what will be the product of the extremes?

15. If the first extreme be 4, what must the other be to make the product 24? How do you find it?

16. If one of the extremes be 8, what must be the other, to make the product 24?

17. If, in a proportion, the product of the two means be 36, what will be the product of the extremes?

18. If one of those extremes be 4, what must be the other? How do you find it?

19. If one of those extremes be 6, what must be the other? How do you find it?

20. If one of those extremes be 12, what must be the other?

DEFINITIONS.

1. RATIO is the comparison of two numbers or quantities with one another.

2. Ratio is written either by fractions, or by separating the numbers by two dots, as $\frac{2}{5}$, or 2 : 5.

3. When two numbers are put together for the purpose of showing their ratio, they are called a *couplet*.

4. The *first term* in a ratio is the *denominator*, when written as a fraction; and it is called the *antecedent*.

5. The *second term* in a ratio is the *numerator*, when written as a fraction; and it is called the *consequent*.

6. When two or more couplets have the same ratio, they are said to be *proportional to one another*. And when arranged together they are called a *proportion*.

7. A proportion is written by putting four points between the two couplets; as, 4 : 8 :: 5 : 10.

8. The *extremes* of a proportion are the first term and the last term.

9. The *means* of a proportion are the two middle terms.

10. One result of an equality of ratios in a proportion, is, that *the product of the extremes is equal to the product of the means*.

11. Hence, if one of the extremes be wanting, we know that it is a number that is a factor of the product of the two means. And because we have the other factor in the other extreme, the factor that is wanting is easily found.

12. The operation of finding the fourth term, or the last extreme, has been called

THE RULE OF THREE.

In the Rule of Three, there are two kinds of proportion, *Direct* and *Inverse*.

When more requires more, or less requires less, the proportion is *direct*.

When more requires less, or less requires more, the proportion is *inverse*.

Rule.

First, Remember that the answer sought will be the last term; which is the consequent of the last couplet. Therefore set down the antecedent of this couplet first.

Second, Consider whether the proportion is direct or inverse.

Third, In writing the two terms of the first couplet, *if the proportion is direct*, the term of the demand must be the *consequent*. But *if the proportion is inverse*, the term of the demand must be the *antecedent*.

Fourth, Find the product of the means by multiplying together the second and third term. Then divide that product by the first term, and the quotient will be the last term or *answer*.

☞ The first two terms must be of the same denomination; and if either of them has several denominations, they must both be brought to the lowest that is mentioned. And, if the last term has several denominations, it must be reduced to the lowest.

Note.—The method in which sums in the Rule of Three have generally been stated, is to make the given terms the first two; and that concerning which the answer is required, the third. Thus, 8 yds. : $6 :: 4 yds. : ans. Then, if the third term is greater than the first, and requires a greater answer; or smaller than the first, and requires a smaller answer; the proportion is said to be *direct*. But, if the third term is greater than the first, and requires a smaller answer; or smaller than the first, and requires a greater answer; the proportion is said to be *inverse*.

In Direct Proportion, the operation is the same as here recommended; but in Inverse Proportion, the product of the first terms is divided by the last term.

APPLICATION.

1. Of a piece of cloth containing 30 yds. I have now 20 yds. If the whole piece was worth $150, what is that worth which I have now?

Ans. *The answer will be in dollars. And as the answer is the consequent of the last couplet, we know that its antecedent also must be in dollars. Therefore we first write down* $150, *as the first term of the last couplet:* thus, : : $150.

We also know that the proportion is direct, because less cloth requires less price; and on this account, the term of demand must be the second term of the first couplet. We therefore write the first couplet in its natural manner before the last term: as follows,

The product of the two means is 3000; which, divided by the first extreme, gives $100 for the last extreme; which is the answer.

yds. yds. $
30 : 20 :: 150
20
3,0)3000
100

2. If $8 buy 4 yds. of cloth, how many yards will $32 buy?

☞ As more dollars will buy more yards, the proportion is direct. Ans. 16 yds.

3. If $48 buy 32 lbs. of tea, how many pounds will $12 buy? Ans. 8 lbs.

4. If 32 lbs. of tea cost $48, what is the cost of 8 lbs.? Ans. $12.

5. If $12 buy 8 lbs. of tea, how much will $48 buy? Ans. 32 lbs.

6. If 8 lbs. of tea cost $12, what is the cost of 32 lbs.? Ans. $48.

7. If 20 yds. of cloth cost 18£., what will 5 yds. cost?
Ans. 20 yds. : 5 yds. :: 18£. : ans.
18£. × 5 = 90£. and 90£. ÷ 20 = 4£. 10s.

8. If 1 C. 74 lbs. of sugar cost 3£. 4s. 6d., what will 9 oz. cost? Ans. 1 C. 74 lbs. : 9 oz. :: 3£. 4s. 6d. : ans.
1 C. 74 lbs. = 2784 oz. and 3£. 4s. 6d. = 744d. Therefore 2784 oz. : 9 oz. :: 774d. : 2½d. nearly.

9. If a man receive $304 for 19 months' work, how much will he receive for 4 months? Ans. $64.

10. If 4 yds. of cloth cost $21.50, what will 10½ yds. cost at the same rate? Ans. $56.43¾.

11. If 3 lbs. of sugar cost 40 cts., how much must I give for 1 C. of the same? Ans. $13.33⅓.

12. If 3 lbs. of tea cost $4.16, what will 10½ pounds cost? Ans. $14.56.

13. If a pole 10 feet long cast a shadow of 2½ feet, what is the height of a steeple whose shadow is 27½ feet?
Ans. 110 feet.

14. Wishing to know the height of a certain steeple, I measured its shade on level ground, 285 feet. I then erected a staff 3 feet long, and found its shade to be 9 feet. How high was the steeple? Ans. 95 feet.

15. If 12 lbs. of butter cost 15s., what is the cost of 462 lbs.? Ans. 28£. 17s. 6d.

16. If a man receive 3s. 4d. a day, what does he receive in a year? Ans. 60£. 16s. 8d.

17. If a staff 5 feet long cast a shade of 3 feet, on level ground, what is the height of a steeple whose shade is 65 feet? Ans. 108 ft. 4 in.

18. If 8 yds. of cloth cost $3.20, what is the cost of 96 yds. of the same? Ans. $38.40.

19. If 2 C. 96 lbs. of sugar cost 6£. 1s. 8d., what is the cost of 35 C. 25 lbs.? Ans. 72£. 8s. $10\frac{2}{7}\frac{1}{4}$d.

20. If the interest on $100 for a year, is $6, what will be the interest of $438.25 for the same time? Ans. 26.29\frac{1}{2}$.

21. If 8 men will do a piece of work in 24 days, in what time can 16 men do it?

Ans. *As the answer will be in days, we first set down 24 days for the antecedent of the last couplet. Then, as more men will require less time, we know that the proportion is inverse; and that the term of demand must be the first term of the first couplet.* So that the proportion will be

men. men. days.
16 : 8 :: 24 :
Ans. 12 days.

22. If 120 men finished a piece of work in 8 months, how many men could do it in 2 months? Ans. 480 men.

23. How many men in 2 months will do as much work as 24 men could do in 8 months? Ans. 96 men.

24. If a man perform a journey in 18 days by traveling 15 hours a day, in how long time would he perform it, if he should travel 12 hours a day? Ans. 22$\frac{1}{2}$ days.

25. If a board 12 inches long and 12 inches wide make a square foot, how long must it be to make a square foot, if it is 9 inches wide? Ans. 16 inches.

26. How long must a board be that is 3 inches wide, to make 6 square feet? [See sum 25.] Ans. 24 feet.

27. How many yds. of carpeting that is 1 yd. wide, will cover a floor 30 feet long and 18 feet wide? Ans. 60 yds.

28. Suppose I lend a friend $200, for 3 months; how many months may I keep $150 of his money to balance the favor? Ans. 4 months.

29. If 12 men build a wall in 20 days, how many will do it in 8 days? Ans. 30.

30. How many pieces of money of 20s. value, are equal to 240 pieces of 12s. each? Ans. 144.

31. How many yards, 3 quarters wide, are equal to 30 yards of 5 quarters wide? Ans. 50 yds.

CANCELING.

Note.—Supposing we have the proportion

12 : 27 :: 8 to x.

In order to find the answer, we multiply the two means, and divide by the first extreme. This operation may be expressed as follows: $\frac{27 \times 8}{12}$.

Now as a fraction, this quantity may be reduced by dividing by 4 and by 3.

Thus, $\frac{27 \times 8}{12} = \frac{27 \times 2}{3}$ which $= \frac{9 \times 2}{1}$, or 18. Therefore, When the first and *either* of the other given terms can be divided by a common measure, perform the division, and use the quotients instead of the given numbers.

32. If 45 barrels of flour cost 23 dollars, how much will 75 barrels cost?

Ans. *Dividing the two terms by* 5, *we obtain* 9 *and* 15; *and dividing by* 3, *we obtain* 3 *and* 5. *Whence the proportion is* 3 : 5 :: 23 to the answer, which is $38\frac{1}{3}$ dollars.

bbls.	bbls.	$
45	75	23
9	15	5
3	5	115

$38\frac{1}{3}$

33. If 435 men consume 96 barrels of provisions in one year, how many barrels will 2435 men consume in the same time?

Ans. *Dividing the first two terms by* 5, *we obtain* 87 *and* 487; *and dividing the two extremes, we obtain* 29 *and* 32. *Whence the proportion is* 29 : 487 :: 32 : Ans. $537\frac{11}{29}$ barrels.

men. men. bbls.
435 : 2435 :: 96
87 : 487 :: 32
29

34. If 18 barrels of flour cost $162, what is the cost of 12 barrels? Ans. $108.

35. If 12 horses eat 30 bu. of oats in a week, how many bushels will 45 horses eat in the same time? Ans. $112\frac{1}{2}$ bu.

36. If a field will pasture 6 cows 91 days, how long will it pasture 21 cows? Ans. 26 days.

37. If 3 pounds of ginger cost 4s. 6d., what is the cost of 26 pounds? Ans. 1£. 19s.

38. If 48 men can build a certain wall in 24 days, how many men can do it in 192 days? Ans. 6 men.

39. If a garrison of 800 men have provision which will last them 6 months, how many men must leave the garrison that the same supply may last 10 months? Ans. 320.

40. If a wall 7 ft. 8 in. high cast a shadow 5 ft. 4 in., what is the height of a steeple casting a shadow 138 ft. 7 in.? Ans. 10 yds. $2\frac{9}{16}$ in.

41. How many yards of stuff $3\frac{3}{4}$ yards wide, are equal to a piece 17 yards long and $2\frac{1}{4}$ wide? Ans. 199 ft. $3\frac{1}{2}$ nls.

42. If 79 lbs. of tea cost 32£. 11s. 9d., what would 43 lbs. come to at the same rate? Ans. 17£. 14s. 9d.

43. If 15 yds. 3 qrs. of broadcloth cost $147, what will 57 yds. cost? Ans. $532.

44. If a horse eat 3 tons of hay in 7 months, how much would he eat in 12 months? Ans. $5\frac{1}{7}$ tons.

It would be well for the pupil, if he wishes more practice in Proportion, to perform by it the sums in Multiplication of Fractions, pages 111, 112, 133, 183, and 184.

RULE OF THREE WITH FRACTIONS.

45. If $\frac{3}{4}$ yard of cloth cost $\frac{4}{5}$ of a dollar, what is the cost of $4\frac{5}{8}$ yards of the same?

Ans. $4\frac{5}{8} = \frac{29}{6}$; *whence the proportion is the same as* $\frac{3}{4} : \frac{29}{6} :: \frac{4}{5}$: *to the answer. Multiplying the two extremes, we have* $\frac{29}{6} \times \frac{4}{5} = \frac{116}{30}$. *And then dividing by the first term, we have* $\frac{116 \times 4}{30 \times 3} = \frac{464}{90} = \frac{232}{45} = \$5\frac{7}{45}$.

yds.	yds.	$
$\frac{3}{4}$	$4\frac{5}{8}$	$\frac{4}{5}$
$\frac{3}{4}$	$\frac{29}{6}$	$\frac{4}{5}$

$\frac{29}{6} \times \frac{4}{5} = \frac{116}{30} \div \frac{3}{4} = \frac{464}{90}$, which $= \frac{232}{45} = 5\frac{7}{45}$.

Or, more expeditiously, $\frac{\overset{2}{\cancel{4}} \times 29 \times 4}{3 \times \underset{3}{\cancel{6}} \times 5} = \frac{232}{45} = 5\frac{7}{45}$.

46. If $\frac{1}{8}$ of a yard of cloth cost $\frac{4}{5}$ of a dollar, what will $\frac{3}{4}$ of a yard cost? Ans. $5.

47. If $\frac{3}{5}$ of a bushel of wheat cost $\frac{7}{8}$ of a dollar, what will $8\frac{1}{2}$ bushels cost? Ans. $10\frac{11}{80}$.

48. How much flannel that is $\frac{3}{5}$ of a yard wide, will line $5\frac{1}{4}$ yards of cloth which is $1\frac{1}{2}$ yards wide? Ans. $14\frac{3}{8}$ yds.

49. If 14 men can do a piece of work in $13\frac{1}{6}$ days, how many men will do the same in $6\frac{7}{12}$ days? Ans. 28 men.

50. If $\frac{3}{16}$ of a ship cost 273£. 2s. 6d., what is $\frac{5}{32}$ of her worth? Ans. 227£. 12s. 1d.

51. If the penny loaf weighs 7 oz. when a bushel of wheat costs 5s. 6d., what is the bushel worth when the penny loaf weighs $2\frac{1}{2}$ oz. Ans. 15s. $4\frac{4}{5}$d.

52. What quantity of shalloon $\frac{3}{4}$ of a yard wide, will line $7\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yd. wide? Ans. 15 yards.

53. What is the value of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of a pound, at the rate of $\frac{9}{10}$ of a dollar for $\frac{8}{9}$ of a pound? Ans. $42\frac{3}{16}$ cents.

54. If $\frac{9}{16}$ of a cwt. of sugar cost $\$14\frac{1}{5}$, what will $7\frac{1}{2}$ cwt. amount to? Ans. $\$118.33\frac{1}{3}$.

COMPOUND PROPORTION;

OR,

DOUBLE RULE OF THREE.

INDUCTIVE EXERCISES FOR THE SLATE.

1. If 36 men reap 60 acres in 5 days, how many men must be employed to reap 240 acres in 12 days?

In this sum we will first calculate for the acres; and consider if 36 men reap 60 acres in 5 days, how many men would reap 240 acres in the same number of days. As the answer is to be *men*, we put 36 men for the antecedent of the last couplet. Also, as more acres require more men, the proportion is direct; and will be,

60 acres : 240 acres :: 36 men : $\frac{240}{60}$ of 36 men.

We will then consider if $\frac{240}{60}$ of 36 men reap 60 acres in 5 days, how many men will reap the same acres in 12 days. As *more* days take *less* men, this proportion will be inverse; and will be,

12 days : 5 days :: $\frac{240}{60}$ of 36 men.

By the first proportion, we find that it would take $\frac{240}{60}$ of 36 men; and by the last proportion, that it would take $\frac{5}{12}$ of $\frac{240}{60}$ of 36 men. This by multiplication $= \frac{1200}{1260}$ of 36 men, which $=$ 60 men.

Whence we see that the ratio of 36 men to the answer is equal to the two ratios, $\frac{240}{60}$ and $\frac{5}{12}$, multiplied together. Now, in these two ratios, the numerators are the consequents of the *first couplets* in the above proportions; and the denominators are the antecedents in the same proportions.

Let us combine the two proportions as follows:

Multiplying the third term by the other mean of each proportion, we obtain 43200; *and dividing that product by the first extreme of each proportion, we obtain* 60. *The answer is* 60 men.

60 acres : 240 acres ::
12 days : 5 days :: } 36 men.

720 1200
36
7200
3600

720)43200(60 men. Ans.
4320
0

Or, by canceling.

$$\left.\begin{matrix}60 : 240 :: \\ 12 : 5 ::\end{matrix}\right\} 36 = \frac{\overset{3}{\cancel{60}} \times \overset{20}{\cancel{240}} \times 5}{\underset{5}{\cancel{60}} \times \underset{1}{\cancel{12}}} = \frac{3 \times 20 \times \cancel{5}}{\cancel{5} \times 1} = 3 \times 20 = 60.$$

DEFINITIONS AND RULE.

1. A *compound ratio* is a ratio that is made by multiplying two or more simple ratios together.

2. A *compound proportion* is a proportion that contains a compound ratio.

RULE.

First, Write the number which is of the same kind with the answer, for the third term.

Second, Arrange the other terms, two and two of a kind, as directed in Simple Proportion, for the first and second terms.

Third, Multiply all the second terms together, and their product by the third term; and divide the last result by the product of all the first terms.

APPLICATION OF THE RULE.

1. If 56 loaves of bread will last 7 men 14 days, how much will last 3 men 21 days?

Ans. $\frac{56 \times 3 \times 21}{7 \times 14} = \frac{8 \times 3 \times 3}{2}$ = Ans. 36 loaves.

2. If 14 horses eat 56 bushels of oats in 16 days, how many bushels will be sufficient to keep 20 horses 24 days? Ans. 120 bushels.

3. If 40s. will pay 8 men for 5 days' work, how much will pay 32 men for 24 days' work? Ans. 38£. 8s.

4. If a family of 8 persons expend $360 in 9 months, how many dollars will keep a family of 18 persons for 12 months? Ans. $1080.

5. If 5 men make 300 pair of shoes in 40 days, how many men will make 900 pair in 60 days? Ans. 10 men.

6. If 16 men mow 112 acres of grass in 7 days, how many acres can 24 men mow in 19 days? Ans. 456.

7. If the carriage of 8 cwt. 128 miles cost 48s., what must be paid for the carriage of 4 cwt. 32 miles? Ans. 6s.

8. If a footman, when the days are 12 hours long, can travel 240 miles in 12 days; in how many days of 16 hours long can he travel 720 miles? Ans. 27 days.

9. If $240 will defray the expenses of 5 men for 22 weeks, 6 days; how long, at the same rate, will 12 men be spending $360? Ans. 14 weeks, 2 days.

10. If the freight of 10 hogsheads of sugar, each weighing 12 cwt., cost $12, for 50 miles; what must be paid for the freight of 40 boxes of the same, each weighing 3½ cwt., for 150 miles? Ans. $42.

11. How many men in 8 days, can complete a ditch 135 yards long when 16 men can dig 54 yards of it in 6 days? Ans. 30 men.

12. If a regiment of soldiers, consisting of 939 men, consumes 351 barrels of wheat in 168 days, how many soldiers will consume 1404 barrels in 56 days? Ans. 11268 soldiers.

13. If the transportation of 8 cwt. 128 miles, costs $12.80; what must be paid for the transportation of 4 cwt. 32 miles? Ans. $1.60.

14. If 3 lbs. of worsted make 10 yards of cloth of 1 yd. 2 qrs. broad; how many pounds would be wanted to make a piece 100 yds. long and 3 qrs. broad? Ans. 15 lbs.

15. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet wide, in 4 days; in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide?

Proportion. 48 men : 32 men ::
36 feet : 864 feet ::
8 : 6 feet ::
4 : 3 :: } 4 days.

Ans. 36 days.

☞ For canceling in sums of this kind, it may be best to use a perpendicular line instead of a horizontal, thus:

Cancel 4 *and* 4; 8 *and* 32, *writing* 4; 6 *and* 36, *writing* 6; 6 *and* 864, *writing* 144; 48 *and* 144, *writing* 3. *The product of the remaining terms is* 36 *days.*

4 days.
48 | 32, 4
6, 36 | 6, 144, 3
8 | 6
4 | 3
36 days.

16. If a stone dealer receives at the rate of $1.25 a rod that is 1½ foot broad, and 1 foot high; how much must he charge for a pile 30 feet long, 26 feet broad, and 4½ feet high? Ans. 177.27\frac{3}{11}$.

17. If 16 compositors set 150 pages of type, each page consisting of 48 lines, and each line of 50 *m*'s, in 8 days of 10 hours each; how many compositors will be required to set 500 pages of 72 lines each, and 45 *m*'s in a line, in 6 days of 8 hours each? Ans. 45 compositors.

18. If 3 men can dig a ditch 13 rods long, 6 feet wide, and 2½ feet deep, in 12 days; how many days will 5 men require to dig a ditch 32½ rods long, 5 feet wide, and 4 feet deep? Ans. 24 days.

19. If 180 men, working 6 days, each day 10 hours, can dig a trench 200 yards long, 3 yards wide, and 2 yards deep; how many days will 100 men be occupied in digging a trench 360 yards long, 4 wide, and 3 deep, working 8 hours a day? Ans. 48 days, 4 hours, 48 m.

INVOLUTION AND POWERS.

DEFINITIONS.

INVOLUTION teaches the method of finding the *powers* of numbers.

The power of any number is the product that is made by multiplying it by itself one or more times.

That number from which any power is produced, is called *the root* of that power.

Any number is *the first power* of itself. The second power of any number is its *square*. The third power of any number is its *cube*. The fourth power is sometimes called the *biquadrate*; and the fifth, the *sursolid*.

The powers are numbered according to the number of times which the root is used as a factor.

The number that denotes the required power of any root is called the *index* or *exponent*. Thus, 3^2 denotes the square of 3; 4^3 denotes the cube of 4.

Then, any power is found by the following

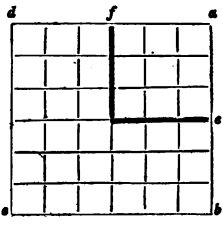
RULE.

Multiply the root, by using it as a factor as many times as there are units in the index of the required power.

APPLICATION.

1. What is the square or 2d power of 15?
   Ans. $15 \times 15 = 225$.
2. What is the cube or 3d power of 6?
   Ans. $6 \times 6 \times 6 = 216$.
3. What is the cube or 3d power of 16? Ans. 4096.
4. How much is 3^3? Ans. 27.
5. How much is 5^4? Ans. 625.
6. What is the cube of 1? Ans. 1.
7. What is the 5th power of 5? Ans. 3125.
8. How much is 464^2? Ans. 215296.
9. How much is $2^3 \times 2^8$? Ans. 2048.

10. What is the square of $\frac{1}{2}$? Ans. $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.



For, take the side of the square figure, and find its half; say from a to e. Make a square on this $a\ e$; and it will reach from a to e, and from a to f, which will embrace $\frac{1}{4}$ of the whole square. For, suppose the whole square to be 6 inches long; then $6\times 6=36$, the area. And as 3 is the half of the side, the square of 3 is 9; and 9 is the $\frac{1}{4}$ of 36. From this we learn that a power of any fraction is always less than its root. But it may be proper to state that the dimensions of the *root* is in *long* measure, but the dimensions of the *power* is in *square* or *solid* measure.

11. What is the cube of $\frac{1}{3}$? Ans. $\frac{1}{27}$.

12. What is the square of $\frac{4}{9}$? Ans. $\frac{16}{81}$.

13. What is the cube of $\frac{3}{4}$? Ans. $\frac{27}{64}$.

14. What is the square of 25.6? Ans. 655.36.

15. What is the cube of .001? Ans. .000000001.

☞ Sometimes, when the index exceeds 3, the calculation may be abridged, by multiplying one power by another. For, to multiply any number by 8, is the same as to multiply it by 2 and 2 and 2 successively. Therefore, to multiply one power by another, will increase the number of the power as many units, as there are units in the index of the power multiplied by. Thus, $4^2\times 4^3=4^5$; and $18^3\times 18^5=18^8$.

16. What is the 6th power of 6? Ans. $6\times 6\times 6=6^3$ or 216; and $216\times 216=6^6$ or 46656.

17. What is the 7th power of 5? Ans. 78125.

18. What is the 8th power of 9? Ans. 43046721.

19. How much is 9^9? Ans. 387420489.

EVOLUTION;

OR,

THE EXTRACTION OF ROOTS.

INDUCTIVE EXERCISES FOR THE SLATE.

$9\times9=81$; and $10\times10=100$. Now, as this is the case; when there are only *two* places of figures in the power, the root is less than 10, and has but *one* figure. $10\times10=100$. Therefore, we know that when there are *three* figures in the power, there must be *two* figures in the square root. Again, as $100\times100=10000$, we know that the moment there are *five* figures in the power, there must be *three* figures in the root. And afterwards, in the same manner, for every two figures that are put in the power, we have one figure in the root.

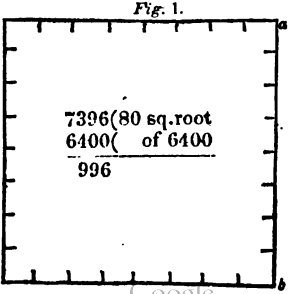
1. Now, how many figures will there be in the square root of the following numbers?

89	36807968	280463289
720	5078947621	1600008943
4367	6048756	189640004578
10047	201345	1467389432100

2. Suppose we have 7396 pieces of stiff paper, each an inch square. We will try to put them together, so as to make a square surface. We know that the greatest square of tens in the number is 6400, which is the square of 80. Therefore, we will first make a square 80 inches long and 80 in. wide.

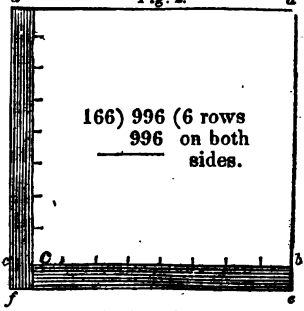
NOTE.—This is represented in figure 1, where we may suppose that each of those divisions, from *a* to *b*, are 10 inches long. This being the case, each side is 80 in. long; and, of course, the square contains 6400 square in. This, subtracted from 7396, leaves 996 blocks to be disposed of.

Fig. 1.



The 996 remaining blocks we will now place on two sides of the square already made, putting the same number on each side, so that when they are all put on, they will still form a square. And as we know that each side is 80 inches long, it will take twice that, or 160, to make one row on both sides. Hence, 996 are enough to make 6 rows on both sides. But by adding 6 rows to the side b, c, we make the side d, e, 6 inches longer, so as to reach on to f. Therefore, the rows on the side d, f, will be 86 inches long. 80 inches + 86 inches = 166 inches, which is the length of one of the rows, b, c, added to one of the rows, d, f. To learn, then, how many blocks will cover both sides, 6 times, we must multiply 166 by 6. $166 \times 6 = 996$, which, when put as directed, keep the square perfect, and take all the blocks.

Fig. 2.



Now, the square at first was 80 inches long; and it has been made 6 inches longer each way. So that the square is now 86 inches long; and $86 \times 86 = 7396$.

When the parts of the operation are put together, it becomes as in the margin. The answer is 80 + 6, or 86.

```
  7396(80
  6400
 ------
  160) 996(6
166×6=996
```

As in division, we may omit the cipher after 8 in the root, and after 64 in the power, provided the cipher be annexed after 16 the double of the root, to make it a true divisor. Then the next figure in the root can be put to the right of the 8 which is already there. Then the sum of that figure, and the divisor, may be written under the divisor. And this sum is to be multiplied by the last figure in the root.

```
  7396(86
  64
 ------
 160)996
 166)996
```

DEFINITION.

EVOLUTION is the operation of finding the root of any power whose quantity is given. The square root is found by the following

RULE.

First, Divide the number into periods of two figures each, by placing a point over every second figure, beginning at the *units'* place.

Second, Find the greatest square in the left hand period, and place it under that period; and put its root in the quotient's place.

Third, Subtract the square that is found, from its period; and to the remainder annex the next period for a new dividend.

Fourth, Suppose a cipher annexed to the root, and then double it, and place it to the left of the dividend for a divisor. Suppose how many times this divisor is contained in the dividend; and place the result on the right of the root, and in the last place of the divisor instead of the cipher.

Fifth, Multiply this divisor by the last quotient figure, and subtract the product from the dividend, as in division; and proceed as before, until all the periods are brought down. The whole quotient will be the square root.

If there be a remainder after all the periods have been brought down, decimals will be obtained, by annexing periods of two ciphers.

The square root of any number may be expressed by placing $\surd$ before that number; thus, $\surd 64 = 8$.

APPLICATION.

1. What is the square root of 55225?

Ans. *The greatest square in* 5 *is* 4, *of which the root is* 2. *Subtracting* 4 *and bringing down the next period, we have* 152. *We have now a block* 20 *square, which will take* 40 *for one row on its two sides; and there will be as many rows as there are* 40 *in* 152, *which is* 3. *But then* 3 *will make the rows on one side* 3 *longer, so that the whole length will be* 43; *and* 3 *times* 43 = 129. *Subtracting* 129 *and bringing down the next period, we have* 2325. *We have now, &c.*

$$
\begin{array}{r|l}
\dot{5}\,5\,\dot{2}\,2\,\dot{5} & 235 \\
4 & \\
\hline
40+3 \;/\; 43 & 152 \\
 & 129 \\
\hline
460+5 \;/\; 465 & 2325 \\
 & 2325 \\
 & =\!=\!=
\end{array}
$$

2. How much is $\surd 2125764$? Ans. 1458.
3. What is the square root of 46.24? Ans. 6.8.
4. What is the sq. root of 102030201? Ans. 10101.
5. How much is $\surd 10342656$? Ans. 3216.
6. How much is $\surd 36372961$? Ans. 6031.
7. How much is $\surd 9712.718051$? Ans. 98.553.
8. What is the square root of 45? Ans. 6.708 +
9. What is the square root of 0.25? Ans. .5.
10. What is the square root of 0.45369? Ans. .673 +
11. What is the square root of .75? Ans. .866 +
12. How much is $\surd .002916$? Ans. .054.

If the fraction be a vulgar fraction, it may be changed to a decimal.

13. What is the square root of $\frac{3}{7}$? Ans. .654 +
14. What is the square root of $\frac{9}{13}$? Ans. .832 +
15. How much is $\surd 7\frac{9}{11}$? Ans. 2.796 +
16. How much is $\surd 3\frac{5}{4\frac{1}{2}}$? Ans. .9128 +
17. How much is $\surd 36\frac{1}{4}$? Ans. 6.0207 +

But if both the numerator and the denominator of a proper fraction, when reduced to its lowest terms, are even powers, find the root of the numerator for a new numerator, and of the denominator for a new denominator.

18. What is the square root of $\frac{1}{4}$? Ans. $\frac{1}{2}$.
19. What is the square root of $\frac{9}{25}$? Ans. $\frac{3}{5}$.
20. What is the square root of $\frac{225}{1024}$? Ans. $\frac{15}{32}$.
21. What is the square root of $\frac{98}{162}$. Ans. $\frac{7}{9}$.
22. What is the square root of $\frac{7056}{9216}$? Ans. $\frac{7}{8}$.

23. A certain square floor is paved with square tiles, of which it contains 5184. How many are on one side? Ans. 72.

24. If an army of 20736 soldiers be drawn up in a perfect square, how many must be placed in rank and file; that is, how long will one rank be? Ans. 144.

EXTRACTION OF THE CUBE ROOT.

1. The cube of 10 is 1000; because $10 \times 10 \times 10 = 1000$. The cube of 100 is 1000000; because $100 \times 100 \times 100 = 1000000$. Therefore, when the power is less than 1000; that is, when it has no more than 3 figures in it; the cube root has but one figure in it. When the cube has as many as four figures and not so many as seven figures, the cube root has two figures in it. And generally, as often as we go beyond a period of three figures in the power, so often we gain one figure in the cube root.

2. What is the cube root of 8? 8000? 8000000?

3. What is the cube root of 27? 27000? 27000000?

4. What is the greatest cube in 37? Ans. 27; because the cube of 4 is 64.

5. Then what is the greatest cube of tens in 37000?
Ans. 27000; which is the cube of 30.

6. What is the greatest cube of tens in 68000?

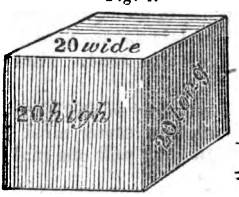
7. What is the greatest cube of tens in 9000?

8. What is the greatest cube of tens in 13000?

Example.

Suppose we have 13824 cubical blocks, each containing one solid inch, which we wish to put into a solid cube, so that the number of blocks in the length, breadth, and thickness shall be equal. We can first pile enough of them to make a solid cube, containing 20 each way. This would take 8000.

Fig. 1.



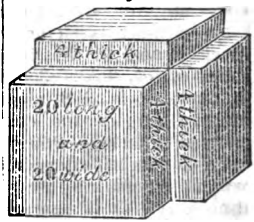
This is represented in the adjoining figure 1, which being 20 blocks or inches long, and 20 wide, requires 400 for one thickness. That multiplied by 20, the number of thicknesses = 8000 blocks or inches. 8000 taken from 13824, leaves 5824 blocks, yet to be disposed of.

20
20
400
20
8000

13824 (20 cube root
8000 of 8000
5824

The remaining 5824 blocks, we can place on three sides of the cube already made, putting the same number on each side, so that when they are all put on, they shall still form a cube. Now, as one side is 20 inches square, it will take 20 times 20, or 400 blocks to cover one side. And, as there are three sides to be covered, it will take 3 times 400, or 1200 blocks, to cover the three sides once. This being the case, 5824 are enough to cover them as many times as there are 1200 in 5824, which is 4 times. And covering it 4 times will take 4 times 1200, that is 4800 blocks.

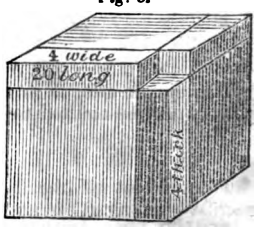
Fig. 2.



Each side 20
20
One surf. 400
3 sides
1200)5824
4+

But, by covering each surface 4 times, we shall find three groove-like places that are not filled; as in figure 2. These places are 20 in. long, 4 in. wide, and 4 in. thick. To find how many blocks it will take to fill one of them, we must multiply its length, width, and thickness together. We find that it will take 320 to fill up one of the grooves; and 3 times 320, or 960 blocks, to fill up the three.

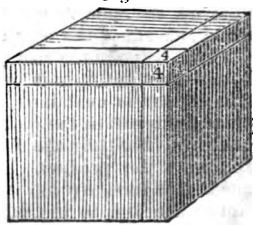
Fig. 3.



20
4
80
4
320
3
960

But, after these 960 blocks are put in their places, we find one corner still incomplete. And it will require a cube 4 in. square, (that is, $4 \times 4 \times 4 = 64$ blocks,) to fill up that corner.

Fig. 4.



4
4
16
4
64

Now, in recapitulation, to cover the three sides 4 times, takes 4 times 1200, or 4800 blocks; and to fill up the three edges, takes 3 times 320, which is 960 blocks; and to fill up the corner, takes 64 blocks. These all amount to 4800 + 960 + 64 = 5824 blocks, just the number that was left after making the first cube. The first cube was 20 inches long; but it has been made 4 inches longer each way. It is now 24 in. long; and 24 × 24 × 24 = 13824.

$$\begin{aligned} 1200 \times 4 &= 4800 \\ 4 \times 4 \times 20 \times 3 &= 960 \\ 4 \times 4 \times 4 &= 64 \\ & 5824 \end{aligned}$$

Hence the operation may be performed as follows.

$$\begin{array}{rl} & 1\dot{3}8\dot{2}\dot{4}\ (\ 20 \\ & 8000 \quad 4 \\ 20 \times 20 \times 3 = 1200\ ) & 5824 \quad 24 \text{ Ans.} \\ & 5824 \end{array}$$

The 5824 is found as follows.

$$\begin{aligned} 4^2 = 16, \text{ and } 16 \times 20 \text{ long} \times 3 \text{ times} &= 4800 \\ 4 \text{ cubed, or } 4^3 &= 960 \\ & 64 \\ & 5824 \end{aligned}$$

The cipher after 2 in the root, and those after 8 in its power, may be omitted as before.

Rule.

First, Point off the figures into places of three figures each, beginning at the unit's place.

Second, Place under the left hand period the greatest cube it contains; and put its root in the quotient's place. Subtract the cube that is found, from its period; and to the remainder annex the next period, for a dividend.

Third, Square the root, after having annexed a cipher to it, and then multiply it by 3, for a divisor. See how often this divisor is contained in the last partial dividend, and place the result in the quotient.

Fourth, Multiply the divisor by this last quotient figure, and note the product. Multiply the square of the last quotient figure, by the other figures in the root with a cipher annexed to them; and multiply that product by 3,

and place the product under the preceding one. Then cube the last figure in the root, and place it under the two ascertained products. Add these three products together, and subtract their sum from the dividend.

Fifth, Bring down the next period to the remainder for a new dividend, and proceed as before, till all the periods are brought down.

APPLICATION.

1. What is the cube root of 18399744?

☞ To be said by the blocks.

18399744 (264 Ans.
8
$20 \times 20 \times 3 = 1200$) 10399
9576
$260 \times 260 \times 3 = 202800$) 823744
823744
0

$1200 \times 6 = 7200$
$6 \times 6 \times 20 \times 3 = 2160$
$6 \times 6 \times 6 = 216$
9576

$202800 \times 4 = 811200$
$4 \times 4 \times 260 \times 3 = 12480$
$4 \times 4 \times 4 = 64$
823744

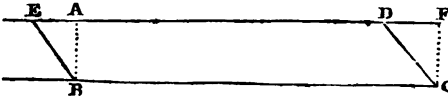
☞ The cube root of any number may be expressed by placing $\sqrt[3]{}$ before that number: thus, $\sqrt[3]{27} = 3$, and $\sqrt[3]{216} = 6$.

2. What is the cube root of 175616? Ans. 56.
3. How much is $\sqrt[3]{}$ 41421736? Ans. 346.
4. How much is $\sqrt[3]{}$ 146363183? Ans. 527.
5. What is the cube root of 16387.064? Ans. 25.4.
6. What is the cube root of 94818816? Ans. 456.
7. What is the cube root of 80.763? Ans. 4.32+
8. What is the cube root of .5? Ans. .793+
9. How much is $\sqrt[3]{}$ .0081? Ans. .2008+
10. What is the cube root of $\frac{3}{13}$? Ans. .61+
11. What is the cube root of $\frac{4}{9}$? Ans. .763.
12. What is the cube root of $\frac{64}{1331}$? Ans. $\frac{4}{11}$.
13. How much is $\sqrt[3]{\frac{21}{4}\frac{9}{9}\frac{7}{1}\frac{3}{3}}$? Ans. $\frac{13}{17}$.
14. What is the cube root of 1627771336? Ans. 546.

TO FIND THE AREA

1. Of a Parallelogram.

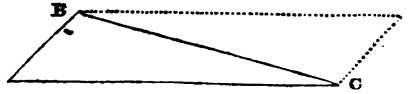
Multiply the length of the base by the perpendicular



which measures the distance from that side to the opposite. Because the triangle E A B will just fill up the space D C F.

2. Of a Triangle.

A triangle is half of a parallelogram. Or it is equal to the product



of the base and ½ the perpendicular, or of the perpendicular and ½ the base.

3. Of a Circle.—Multiply half the circumference by half the diameter.

Examples.

1. In a three-sided field, the longest side is 75 rods, and a perpendicular from that side to the opposite angle, is 50 rods. How many acres does it contain?

Ans. 11 acres, 2 roods, 35 rods.

Note.—The area of any irregular figure may be found by dividing it into triangles, and adding together their several areas.

2. There is a circular fishpond 56 feet in diameter, and 136 feet in circumference. How many square feet does it contain? Ans. 2464.

Note.—The diameter of a circle multiplied by 3.1416, equals the circumference; and the circumference divided by 3.1416, equals the diameter.

3. What is the circumference and area of a circle whose diameter is 20 rods? Ans. 1 acre, 3 roods, 34 rods. +

4. What is the solid contents of a round stick of timber of equal size from end to end, whose diameter is 21 inches, and length 20 feet?

Ans. Area of one end, 2.4052 ft., which × by 20 ft. = 48.104 sol. ft.

THE END.